نسألكم الدعاء

IF you download the Free **APP.** RC Structures YASSER EL LEATHY on your smart phone or tablet, you will be able to play illustrative movies For any paragraph that has a QR code icon

اذا حملت تطبيق RC Structures YASSER EL LEATHY على تليفونك المحمول او اللوح السطحى ستستطيع أن تشغل أفلام شرح للمقاطع التى تحتوى على رمز

Solving Indeterminate Structures. Table of Contents.

Introduction.

في المسائل التى يكون فيها ال system عباره عن Indeterminate system
يفضل أن نحل هذا ال system بطريقه من الطرق الأتيه :

ⓐ Moment Distribution Method.
ⓑ Virtual Work Method.
ⓒ Approximate Method. (Not For all systems).

ⓐ Moment Distribution Method.

We better use it IF there is no Sway.

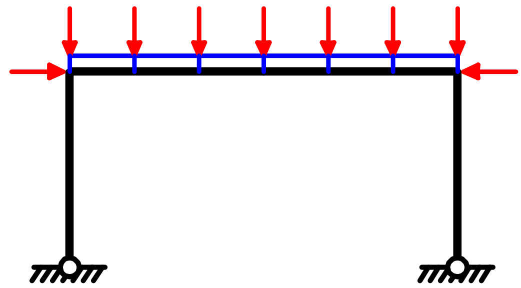
يفضل العمل بهذه الطريقه اذا لم يوجد (sway) على ال (system) .

Systems solved by Moment distribution Method :-

① Two Hinged Frame.

The Frame has to be symmetric in Loads & Dimensions.

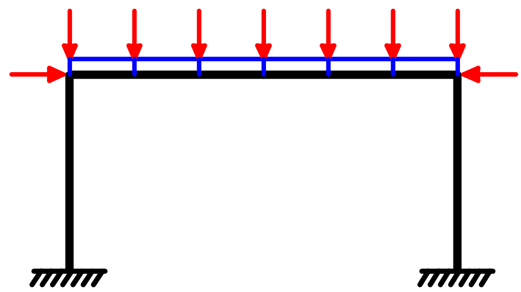
يجب أن يكون متماثل فى الاحمال و الابعاد .



② Fixed Frame.

The Frame has to be symmetric in Loads & Dimensions.

يجب أن يكون متماثل فى الاحمال و الابعاد .



اذا وجد (sway) على ال (system) و أردنا حله بطريقه Moment distribution
يجب أن نعمل sway correction و ذلك سيكون صعب فى الحل .
لذا الاسهل فى هذه الحاله الحل بطريقه Virtual work .

ⓑ Virtual Work Method.

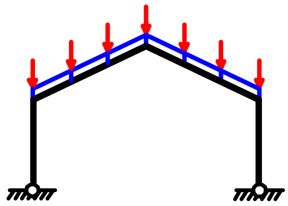
We better use this method IF:

① IF there is a Sway.

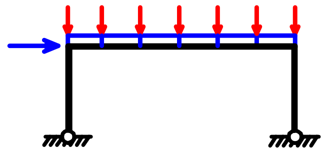
② IF there is a link member.

Systems solved by Virtual Work Method :-

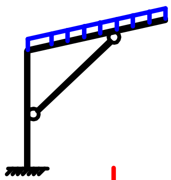
① Two Hinged Inclined Frame.



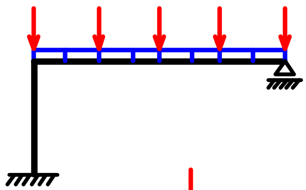
② Two Hinged Frame subjected to HL. Load From one Side.



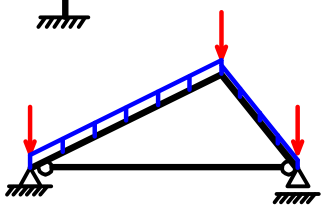
③ Cantilever Frame with Link member.



④ Fixed–Roller Frame



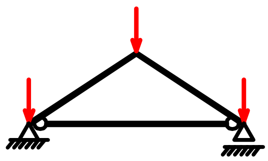
⑤ Saw Tooth (Girder type).



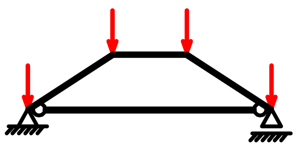
ⓒ Approximate Method.

Systems solved by Approximate Method :-

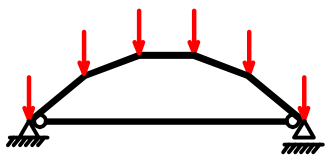
① Triangular Polygon Frame.



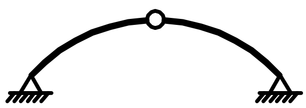
② Trapezoidal Polygon Frame.



③ Arch Girder.



④ Arch Slab.



① Moment Distribution Method.

ⓐ Get Moment of Inertia For all members. 《 I_c , I_b 》

ⓑ Calculate the stiffness For each member. 《 K_c , K_b 》

ⓒ Get Distribution Factors at all Joints. 《 D.F. 》

ⓓ Get Fixed End Moments For Beams. 《 F.E.M. 》

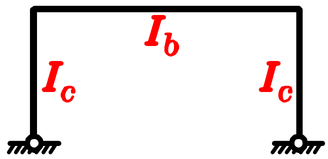
ⓔ Get the Final Moment. 《 $M_F = F.E.M._{(beam)} * D.F._{(col.)}$ 》

ⓕ Get B.M.D. , N.F.D. , S.F.D.

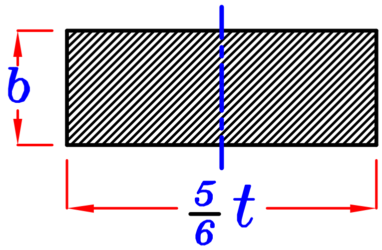
ⓐ Get Moment of Inertia For all members. 《 I_c , I_b 》

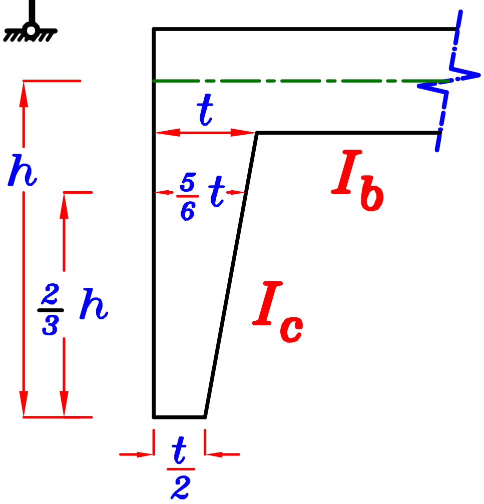
* For Column.

① Two Hinged Frame.

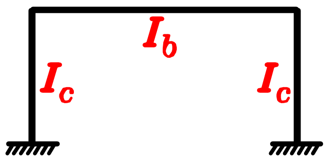


$$\boxed{I_c = \frac{b\left(\frac{5}{6}t\right)^3}{12}}$$

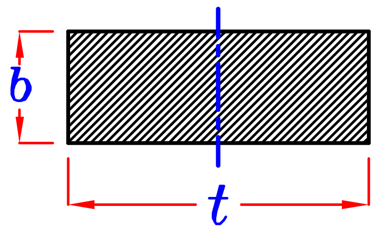


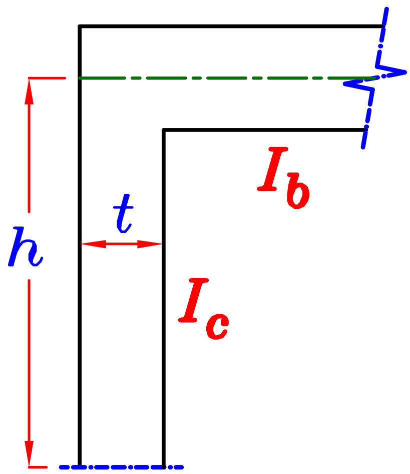


② Fixed Frame.

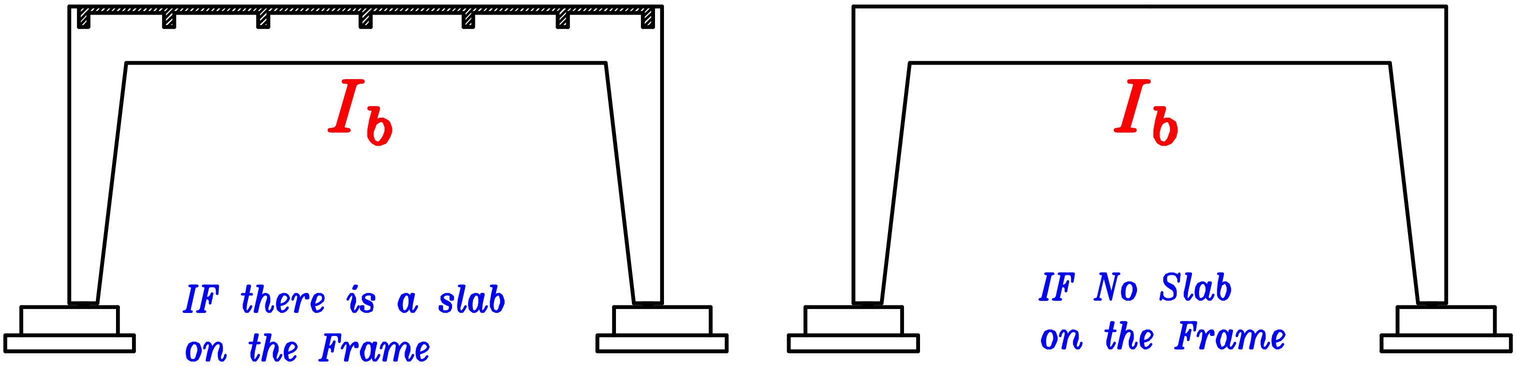


$$\boxed{I_c = \frac{b(t)^3}{12}}$$





* **For Beams.**



① IF there is a slab on the Frame.

$$\boxed{I_b = (\mu * 10^{-4})\, B\, t^3}$$

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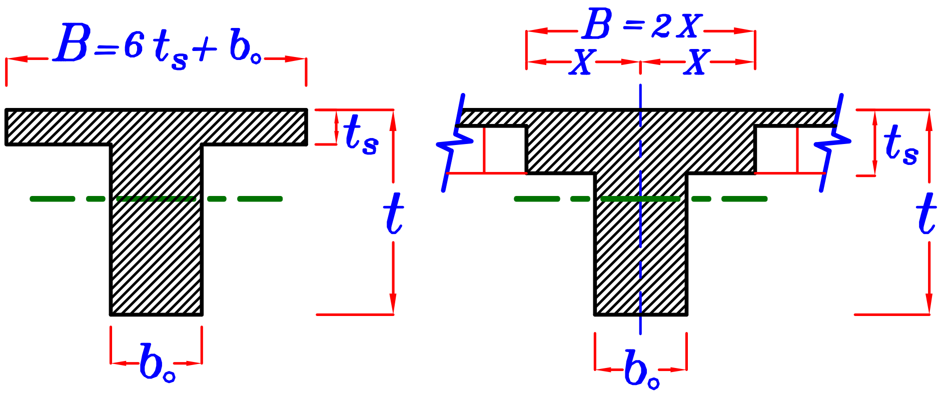
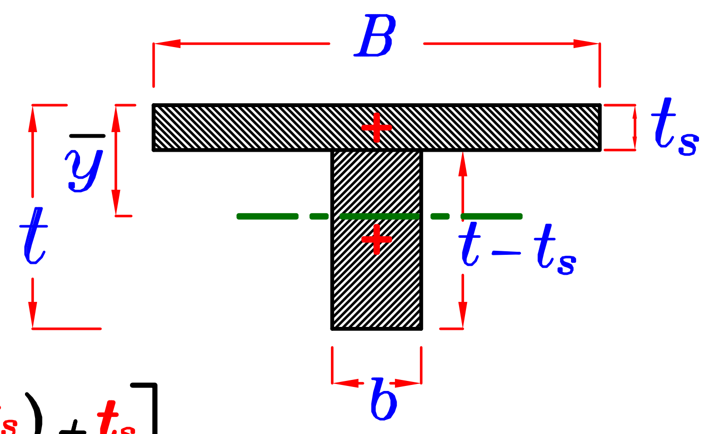


For Solid slab. For H.B. slab.

أو ممكن حساب الـ I_b كالاتى



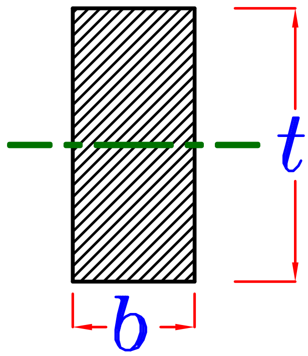
$$A = B\,t_s + b\,(t - t_s)$$

$$\bar{y} = \frac{B\,t_s\left(\frac{t_s}{2}\right) + b\,(t - t_s)\left[\left(\frac{t - t_s}{2}\right) + t_s\right]}{A}$$

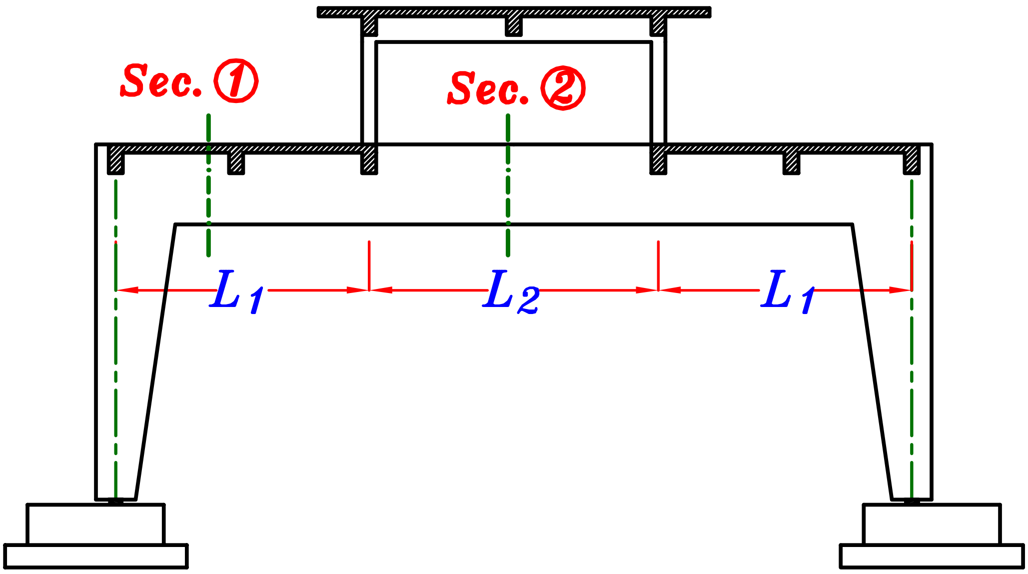
$$I_b = \frac{b\,(t - t_s)^3}{12} + b\,(t - t_s)\left(\left(\frac{t - t_s}{2}\right) + t_s - \bar{y}\right)^2 + \frac{B\,t_s^3}{12} + B\,t_s\left(\bar{y} - \frac{t_s}{2}\right)^2$$

② IF there is no slab on the Frame.

$$I_b = \frac{b(t)^3}{12}$$



ملحوظه



عند وجود شخشيخه يوجد قطاعان فى كمره ال **Frame**

Sec. ① I_{b1} Sec. ② I_{b2}

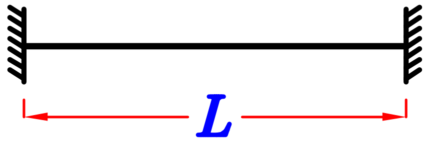
$$I_b = \frac{I_{b1} * 2L_1 + I_{b2} * L_2}{2L_1 + L_2}$$

نأخذ المتوسط

ⓑ Calculate the stiffness For each member. (K_c, K_b)

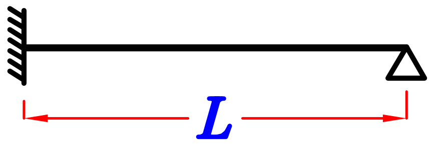
① **Fixed–Fixed**

$$K = \frac{I}{L}$$



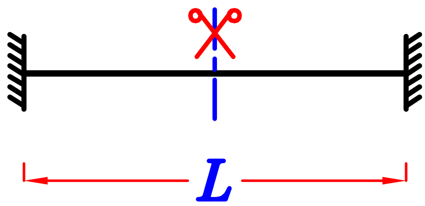
② **Fixed–Hinged**

$$K = \frac{3}{4}\,\frac{I}{L}$$



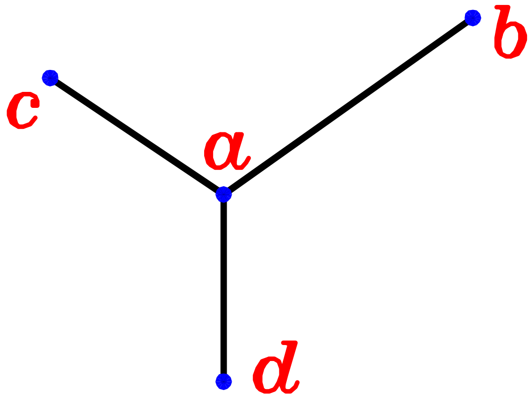
③ **Fixed–Fixed** (Symmetric)

$$K = \frac{1}{2}\,\frac{I}{L}$$



ⓒ Get Distribution Factor For all Joints.

For any Joint



$$\sum K = K_{(a-b)} + K_{(a-c)} + K_{(a-d)}$$

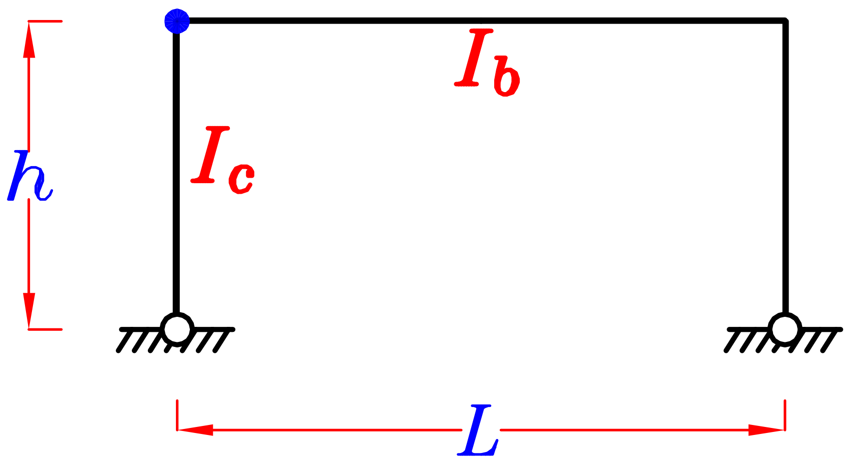
$$D.F._{(a-b)} = \frac{K_{(a-b)}}{\sum K}$$

$$D.F._{(a-c)} = \frac{K_{(a-c)}}{\sum K}$$

$$D.F._{(a-d)} = \frac{K_{(a-d)}}{\sum K}$$

Note: For any Joint $\sum D.F. = 1.0$

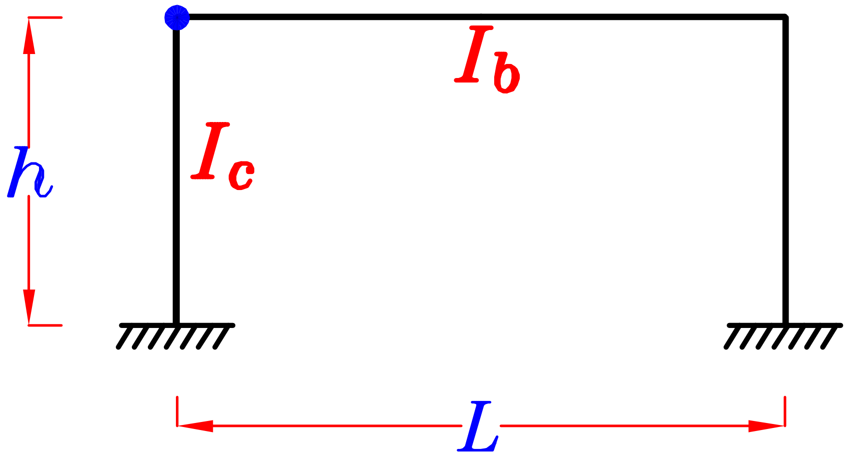
① Two Hinged Frame.



$$K_b = \frac{1}{2}\,\frac{I_b}{L} \qquad K_c = \frac{3}{4}\,\frac{I_c}{h}$$

$$D.F._b = \frac{K_b}{K_b + K_c} \qquad D.F._c = \frac{K_c}{K_b + K_c}$$

② Fixed Frame.



$$K_b = \frac{1}{2}\,\frac{I_b}{L} \qquad K_c = \frac{I_c}{h}$$

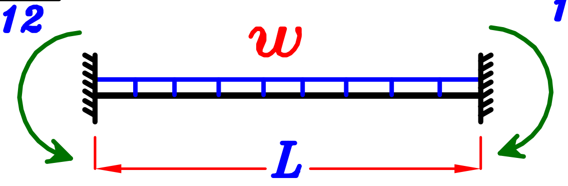
$$D.F._b = \frac{K_b}{K_b + K_c} \qquad D.F._c = \frac{K_c}{K_b + K_c}$$

ⓓ Get Fixed end Moments For Beams. « F.E.M. »

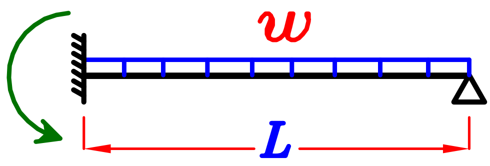
إذا كان العزم يدور فى نفس إتجاه عقارب الساعه تكون الأشاره $(+Ve)$ $+Ve$

إذا كان العزم يدور فى إتجاه عكس عقارب الساعه تكون الأشاره $(-Ve)$ $-Ve$

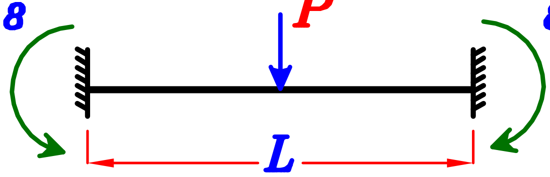
$-\dfrac{wL^2}{12}$ $+\dfrac{wL^2}{12}$



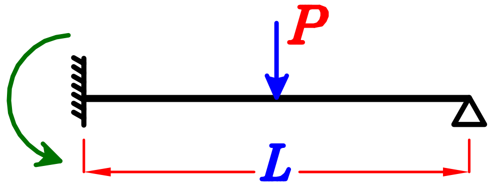
$-\left[\dfrac{wL^2}{12}+\dfrac{1}{2}\,\dfrac{wL^2}{12}\right]=-\dfrac{wL^2}{8}$



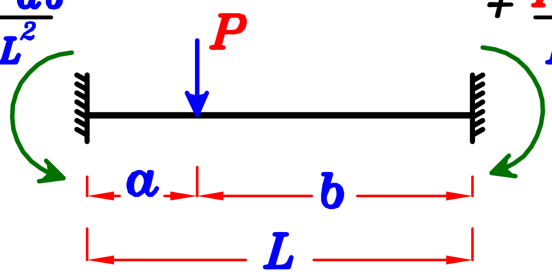
$-\dfrac{PL}{8}$ $+\dfrac{PL}{8}$



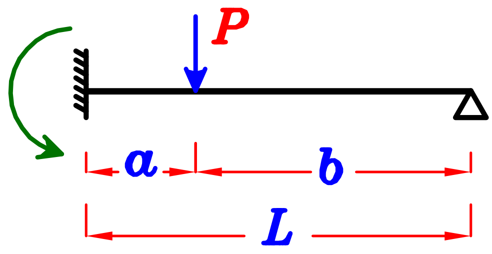
$-\left[\dfrac{PL}{8}+\dfrac{1}{2}\,\dfrac{PL}{8}\right]=-\dfrac{3PL}{16}$



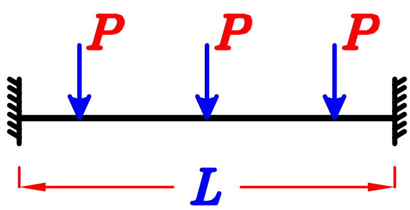
$-\dfrac{Pab^2}{L^2}$ $+\dfrac{Pba^2}{L^2}$



$-\left[\dfrac{Pab^2}{L^2}+\dfrac{1}{2}\,\dfrac{Pba^2}{L^2}\right]$

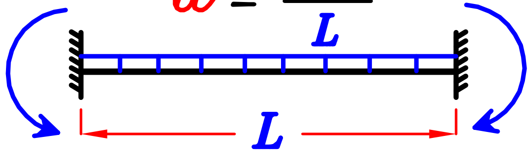


ممكن للتسهيل عمل حل تقريبى و هو تحويل الاحمال المركزه الى حمل منتظم
مع شرط أن يكون عدد الاحمال المركزه على الكمره لا يقل عن ٣



IF number of Loads $\geqslant 3$ →

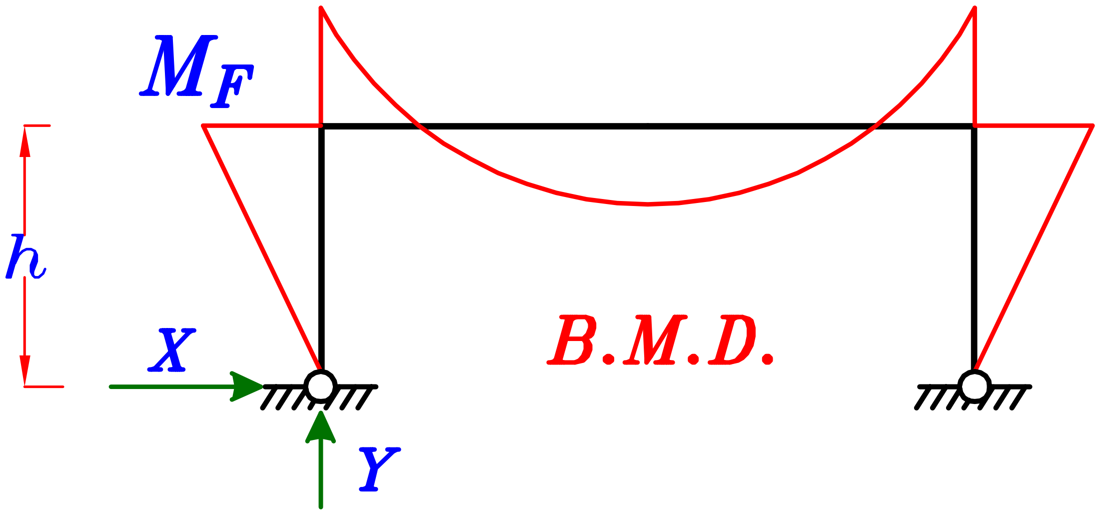
$-\dfrac{wL^2}{12}$ $w=\dfrac{\Sigma P}{L}$ $\dfrac{wL^2}{12}$



ⓔ Get the Final Moment. ((M_F))

بدل عمل جدول لل *moment distribution* يتم حساب العزم الخارجي للعمود M_F

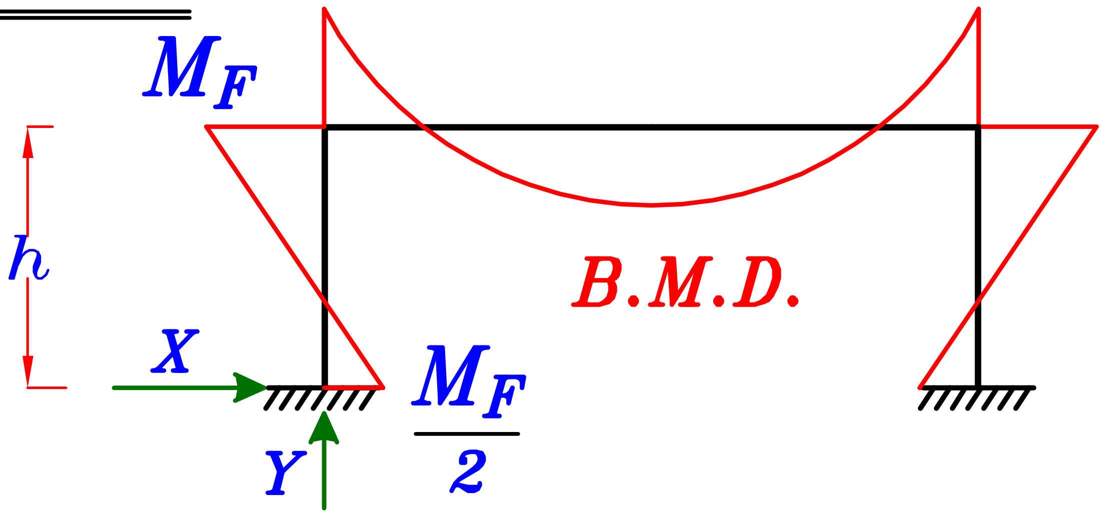
① Two Hinged Frame.



$$M_F = F.E.M._{(beam)} * D.F._{(col.)}$$

$$X = \frac{M_F}{h} \quad , \quad Y = \frac{\Sigma\, Load}{2}$$

② Fixed Frame.

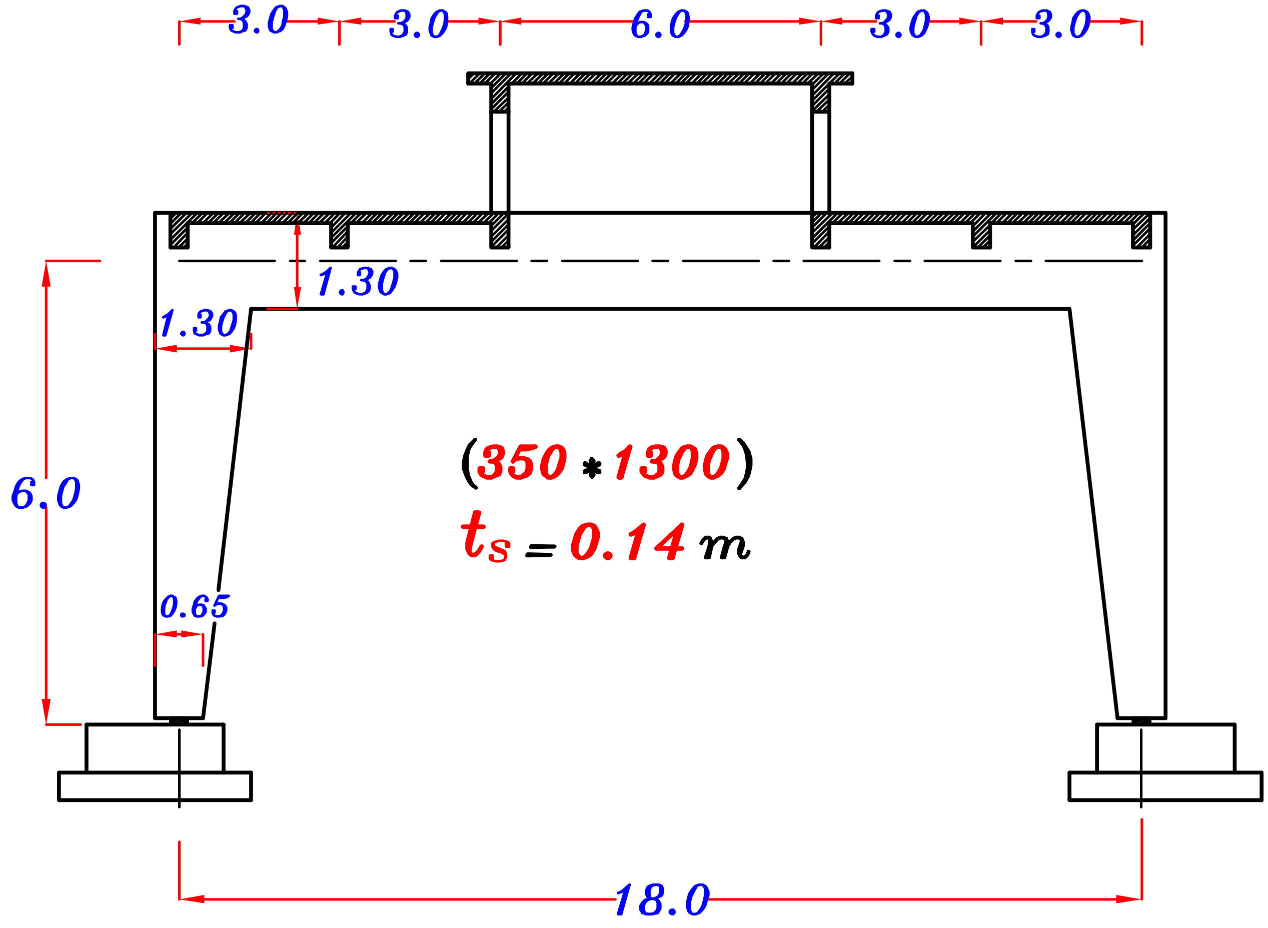


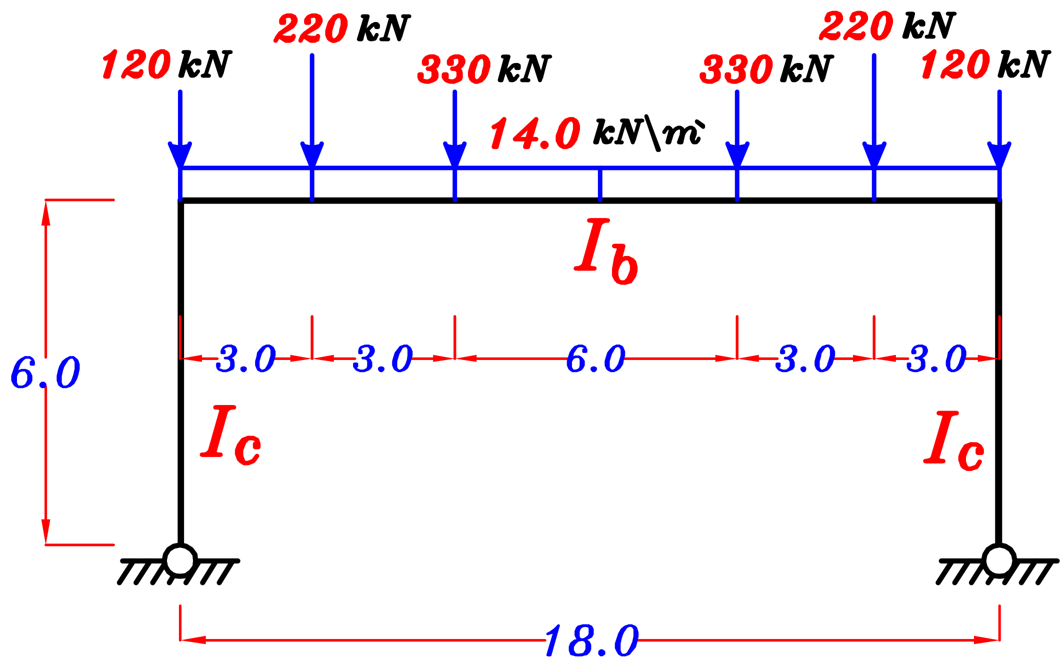
$$M_F = F.E.M._{(beam)} * D.F._{(col.)}$$

$$X = \frac{M_F + M_F \backslash 2}{h} \quad , \quad Y = \frac{\Sigma\, Load}{2}$$

Two Hinged Frame.

Example.





For the given Frame, Draw B.M.D. & N.F.D.

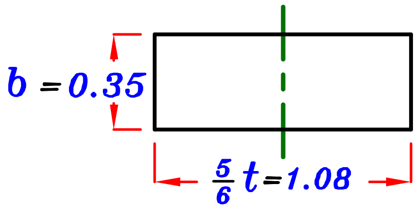
For the Two hinged Frame

we will use Moment Distribution Method.

Solution.

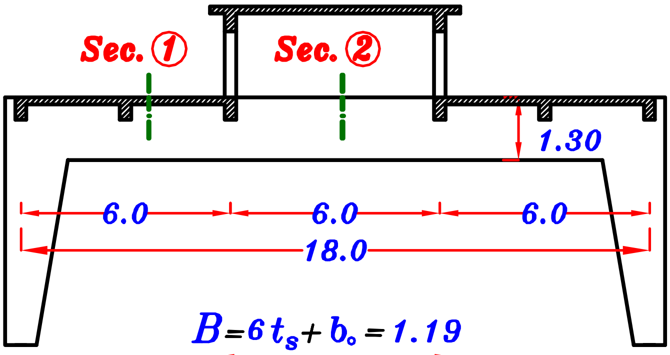
@ Get Moment of Inertia For all members. « I_c , I_b »

I_c



$$I_c = \frac{b\left(\frac{5}{6}t\right)^3}{12} = \frac{0.35\left(\frac{5}{6} * 1.30\right)^3}{12} = 0.03708 \ m^4$$

I_b



I_{b1}

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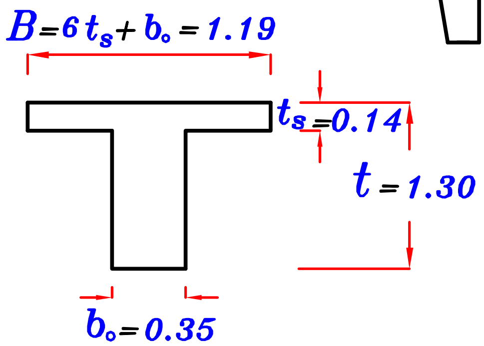
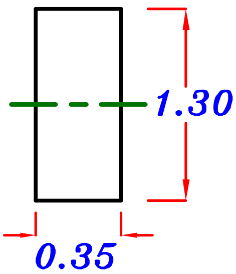


$$\left.\begin{aligned} \frac{t_s}{t} &= \frac{0.14}{1.30} = 0.107 \\ \frac{b_o}{B} &= \frac{0.35}{1.19} = 0.294 \end{aligned}\right\} \mu = 365$$

$$I_b = (\mu * 10^{-4})\, B\, t^3 = 365 * 10^{-4} * 1.19 * 1.30^3 = 0.09542 \ m^4$$

I_{b2}

$$I_{b2} = \frac{b\,(t)^3}{12} = \frac{0.35\,(1.30)^3}{12} = 0.06408 \ m^4$$



I_b

$$\boxed{I_b = \frac{I_{b1} * 2 L_1 + I_{b2} * L_2}{2 L_1 + L_2}}$$ نأخذ المتوسط

$$I_b = \frac{0.09542 * 12.0 + 0.06408 * 6.0}{18.0} = 0.08497$$

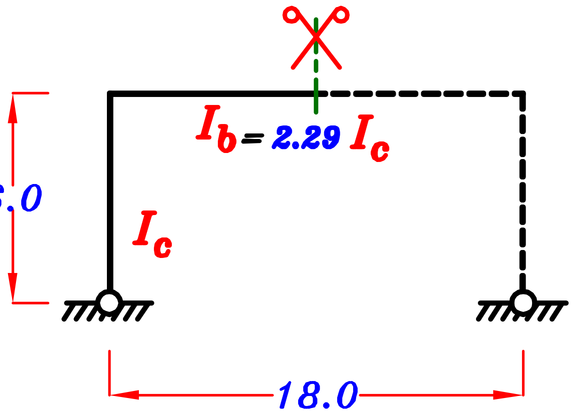
$$\therefore \boxed{I_b = 2.29 \ I_c}$$

ⓑ Calculate the stiffness For each member. « K_c, K_b »

D.F. For Joint **b**

$$K_c = \frac{3}{4}\frac{I_c}{h} = \frac{3}{4} * \frac{I_c}{6.0} = 0.125 I_c$$

$$K_b = \frac{1}{2}\frac{I_b}{L} = \frac{1}{2} * \frac{(2.29) I_c}{18} = 0.0636 I_c$$



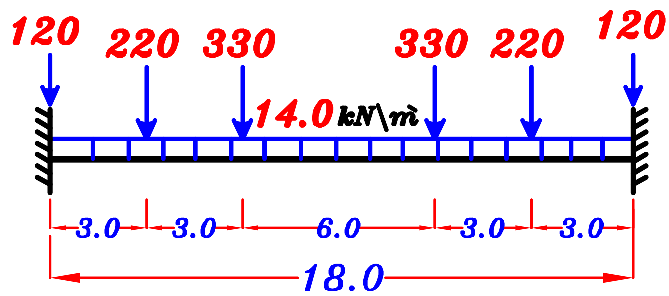
ⓒ Get Distribution Factors at all Joints. « D.F. »

$$D.F._c = \frac{0.125}{0.125 + 0.0636} = 0.663$$

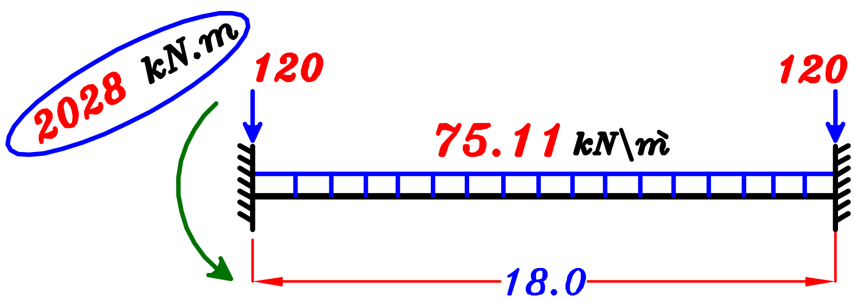
$$D.F._b = 1 - 0.663 = 0.337$$

ⓓ Get Fixed End Moments For Beams. « F.E.M. »

لان عدد الاحمال المركزه لا يقل عن ٣ ممكن تحويل الاحمال الى حمل واحد منتظم .



$$w = o.w. + \frac{\sum P}{span} = 14.0 + \frac{2\,(220) + 2\,(330)}{18.0} = 75.11 \ kN\backslash m'$$



$$F.E.M._{(beam)} = \frac{75.11 * 18^2}{12} = 2028 \ kN.m$$

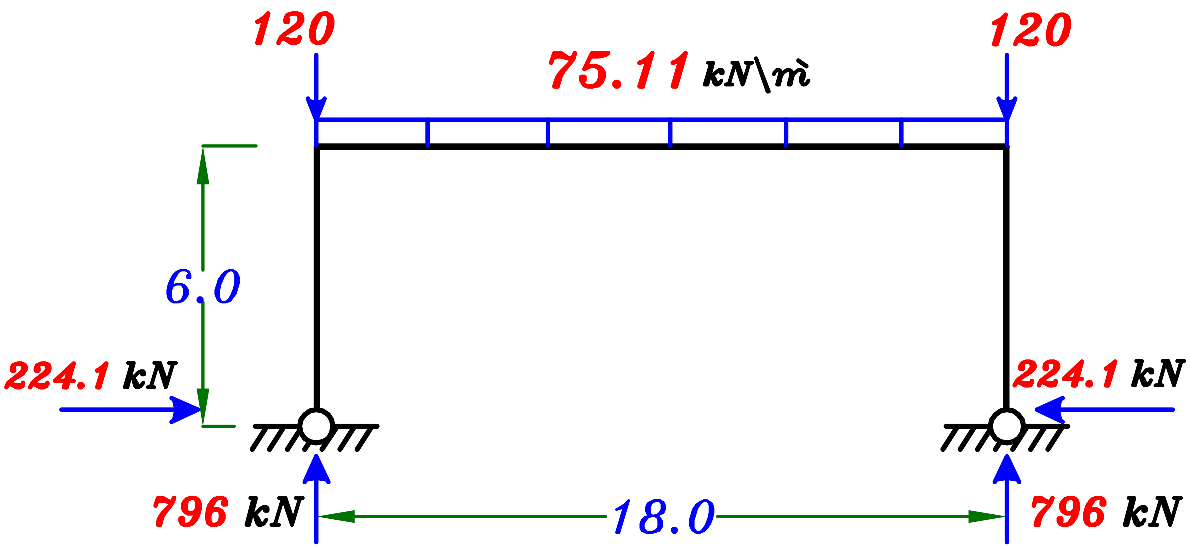
ⓔ Get the Final Moment. ((M_F))

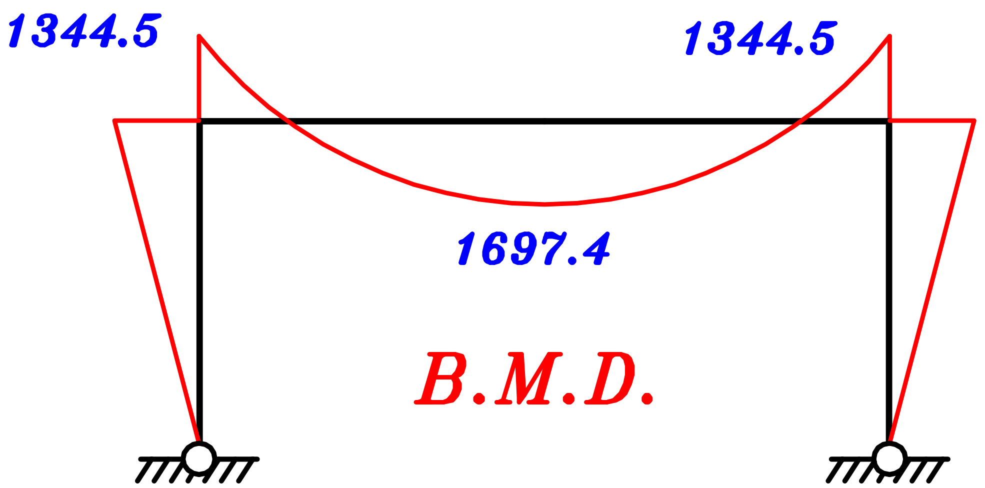
$$M_F = F.E.M._{(beam)} * D.F._{(col.)} = 2028 * 0.663 = 1344.5 \ kN.m$$

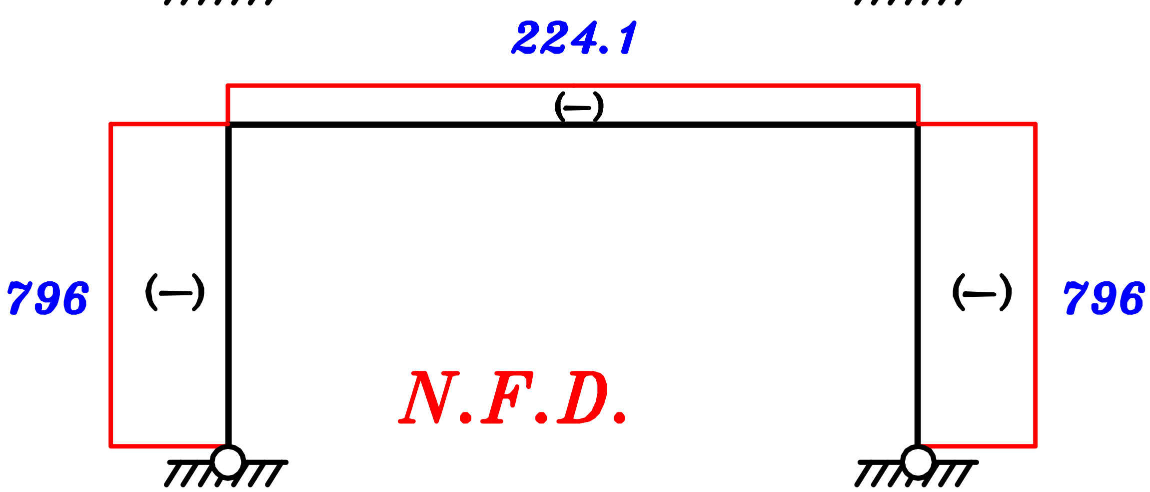
$$X = \frac{M_F}{h} = \frac{1344.5}{6.0} = 224.1 \ kN$$

$$Y = \frac{\Sigma \ Load}{2} = \frac{75.11 * 18 + 2 * 120}{2} = 796 \ kN$$

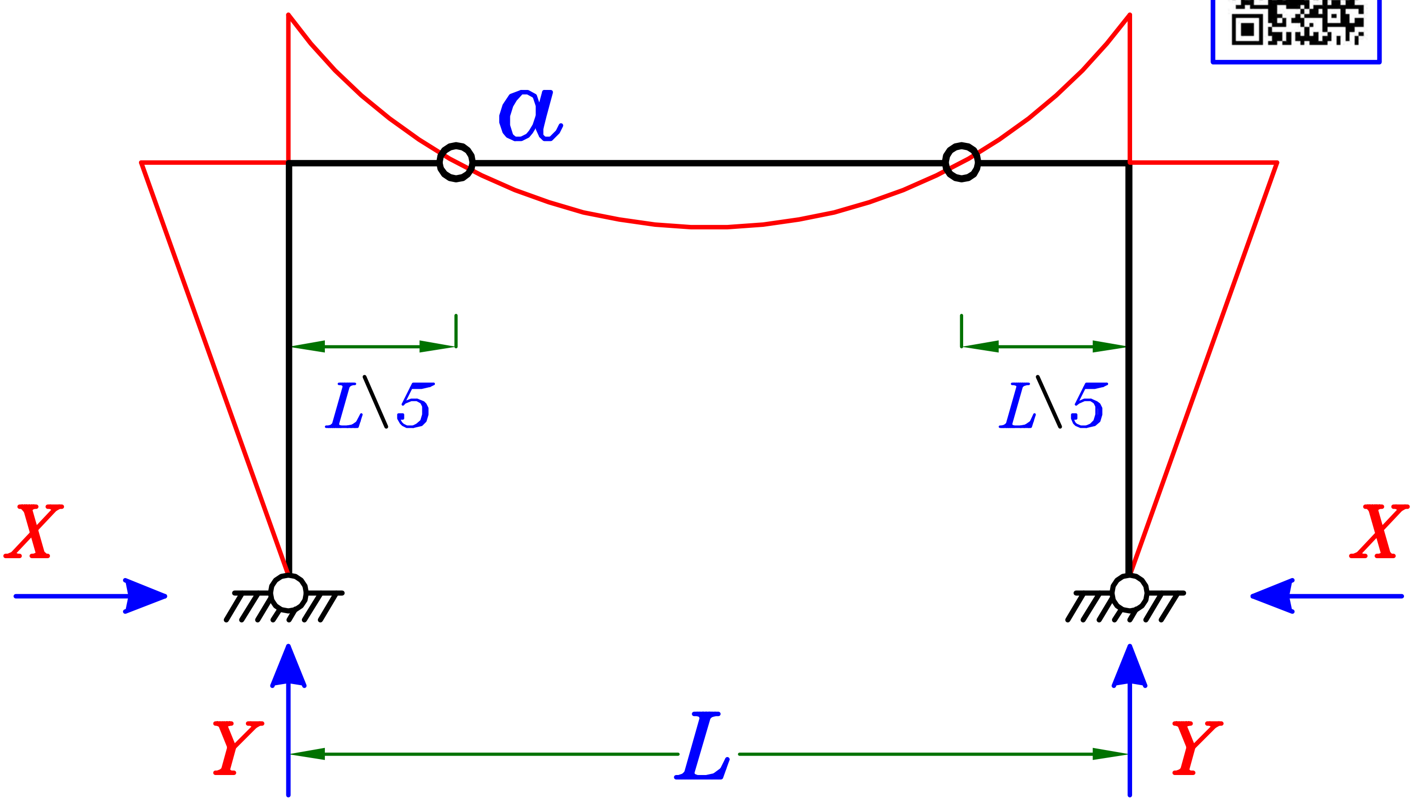
ⓔ Get B.M.D. , N.F.D.







Approximate Solution.



assume that in the beam there is an intermediate hinge at $\frac{L}{5}$

$$Y = \frac{\sum Loads}{2}$$

To get the reactions X

Take the moment at Point a = Zero

Then Draw Internal Forces Diagrams.

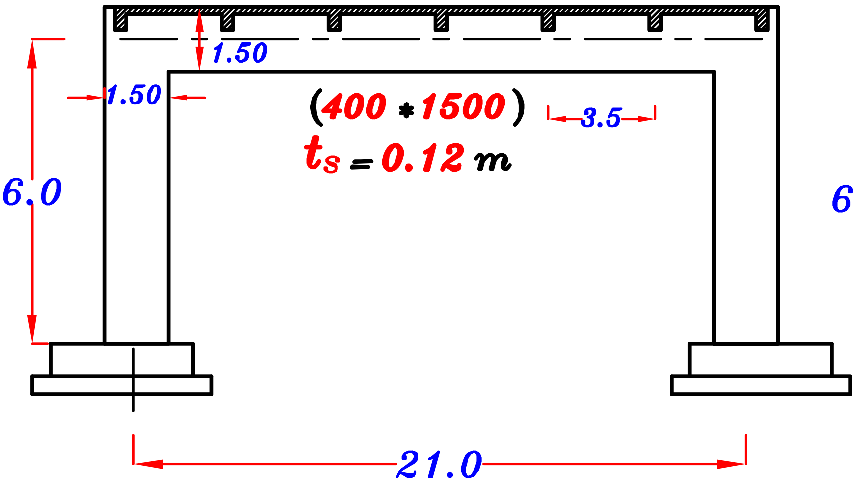
ملحوظه هامه

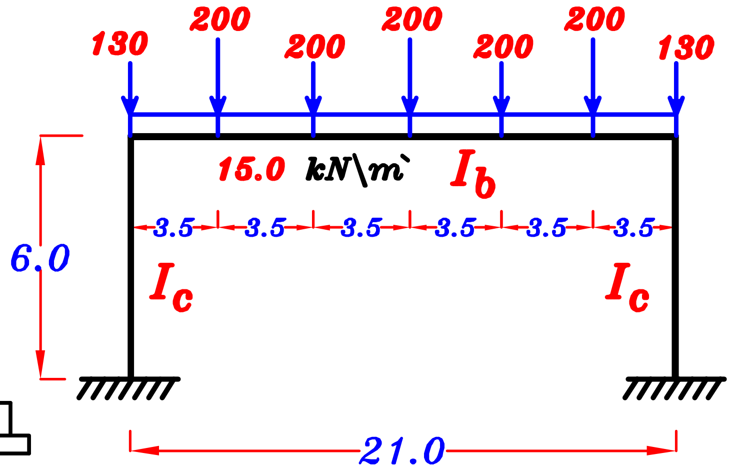
هذا الحل حل تقريبى جدا و غير دقيق ، لذا لن نستخدم هذا الحل الا مع تعذر الوقت فى الامتحان .

Fixed Frame.

Example.

For the given Frame, Draw B.M.D. & N.F.D.





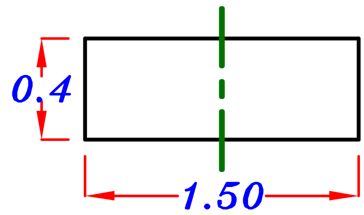
For the Fixed Frame
we will use Moment Distribution Method.

Solution.

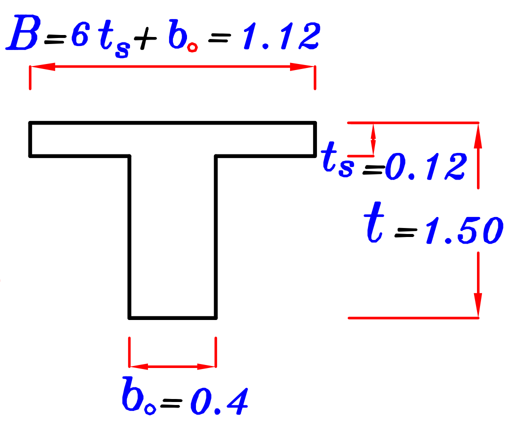
@ Get Moment of Inertia For all members. ((I_c , I_b))

I_c

$$I_c = \frac{b(t)^3}{12} = \frac{0.4\,(1.50)^3}{12} = 0.1125 \ m^4$$



I_b



$$\left.\begin{aligned} \frac{t_s}{t} &= \frac{0.12}{1.50} = 0.08 \\ \frac{b_\circ}{B} &= \frac{0.4}{1.12} = 0.357 \end{aligned}\right\} \quad \text{Table page 63} \quad \mu = 392$$

$$I_b = (\mu * 10^{-4})\, B\, t^3 = 392 * 10^{-4} * 1.12 * 1.50^3$$

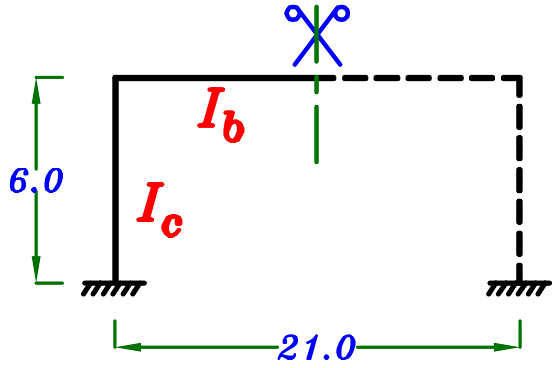
$$= 0.148 \ m^4$$

$$\therefore \quad \boxed{I_b = 1.317\, I_c}$$

ⓑ Calculate the stiffness For each member. « K_c , K_b »

$$K_c = \frac{I_c}{h} = \frac{I_c}{6.0} = 0.167\ I_c$$

$$K_b = \frac{1}{2}\,\frac{I_b}{L} = \frac{1}{2} * \frac{(1.317)I_c}{21} = 0.0313\ I_c$$



ⓒ Get Distribution Factors at all Joints. « D.F. »

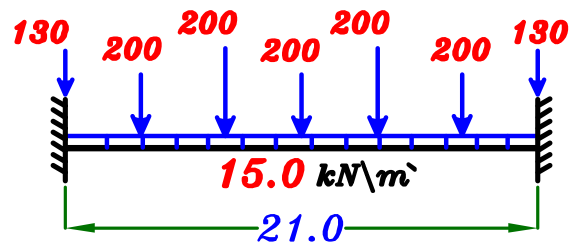
$$D.F._{c} = \frac{0.167}{0.167 + 0.0313} = 0.842$$

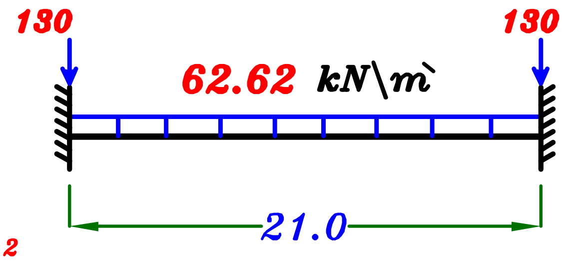
$$D.F._{b} = 1 - 0.842 = 0.158$$

ⓓ Get Fixed End Moments For Beams. « F.E.M. »

$$w = o.w. + \frac{\sum P}{span}$$

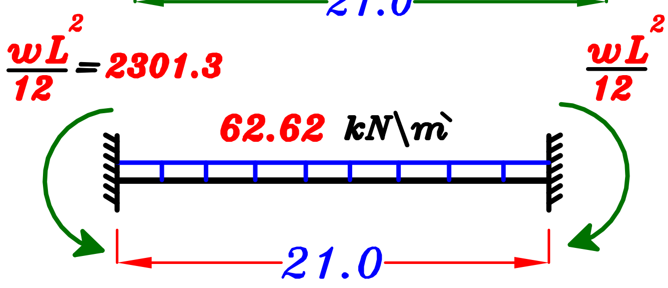
$$= 15.0 + \frac{5\,(200)}{21.0} = 62.62\ kN\backslash m`$$





F.E.M.

$$\frac{wL^2}{12} = \frac{62.62 * (21.0)^2}{12} = 2301.3\ kN.m$$

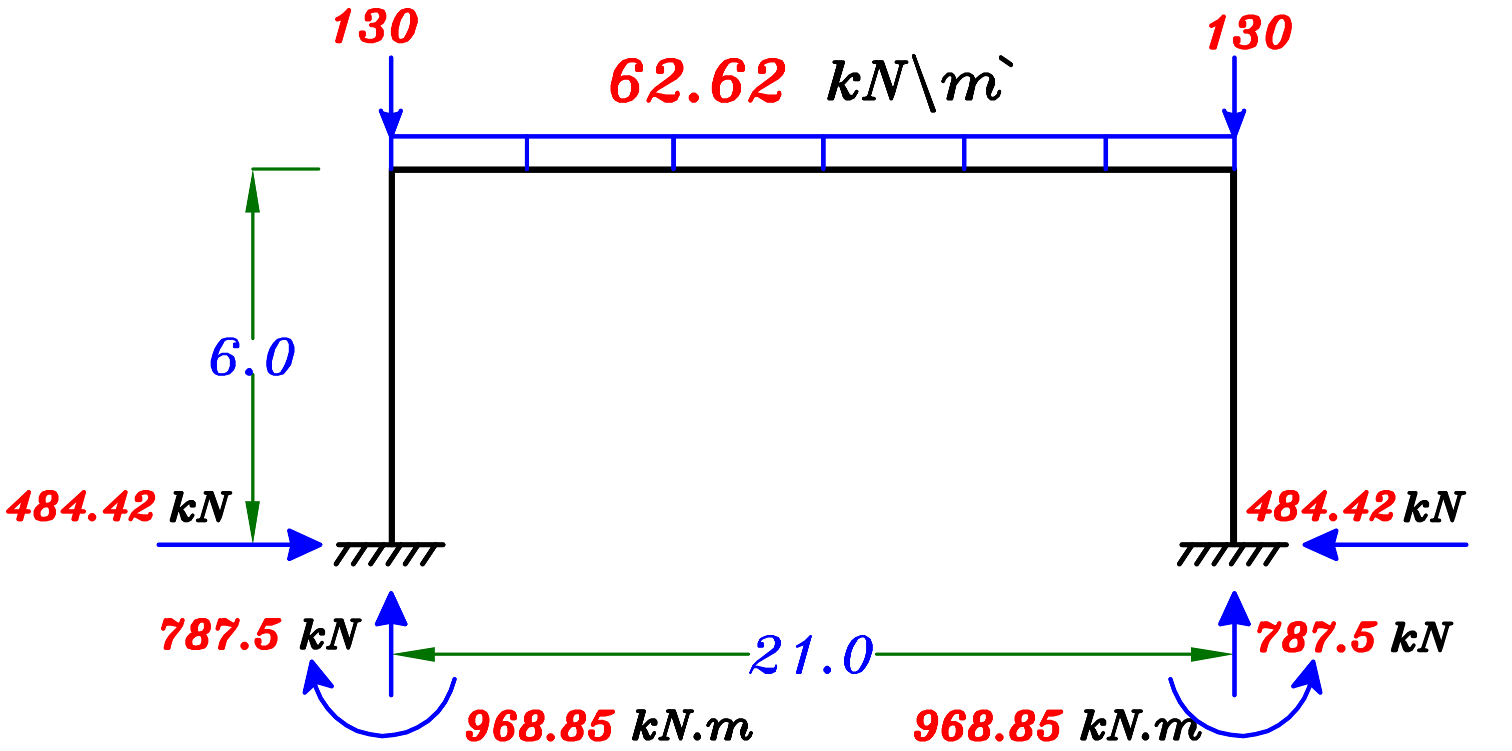


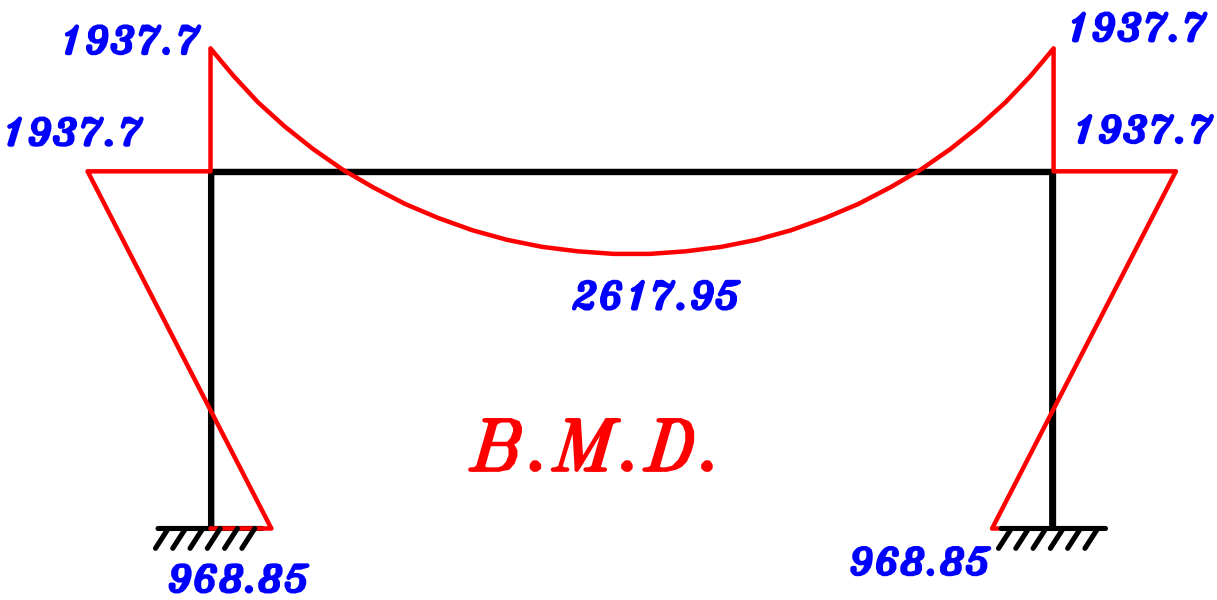
ⓔ Get the Final Moment. « M_F »

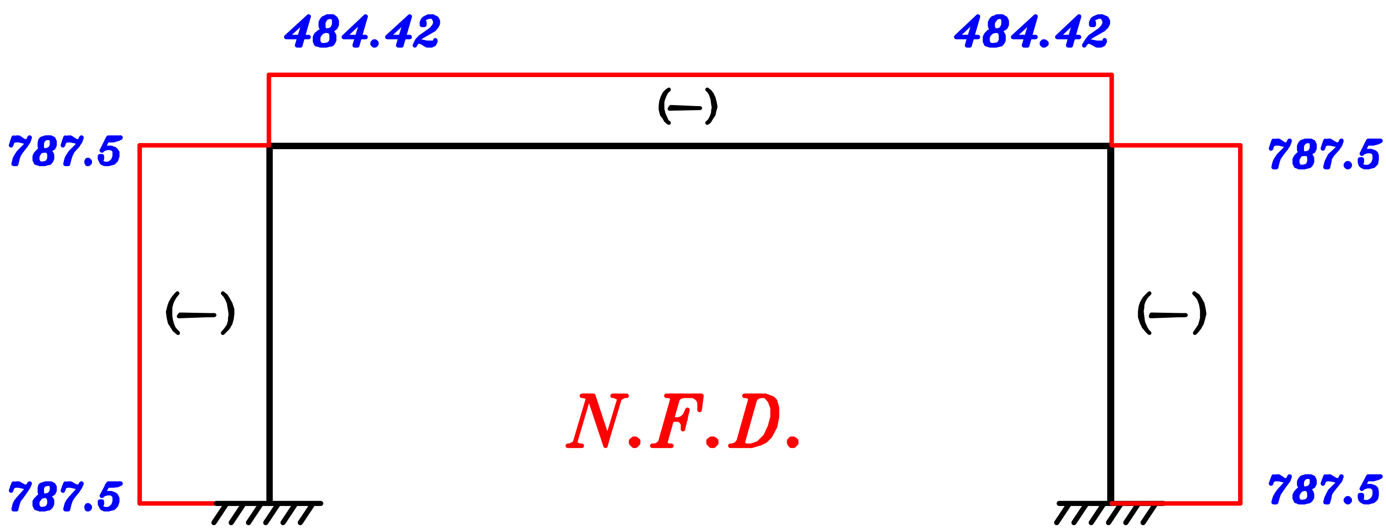
$$M_F = F.E.M._{(beam)} * D.F._{(col.)} = 2301.3 * 0.842 = 1937.7\ kN.m$$

$$X = \frac{M_F + M_F\backslash 2}{h} = \frac{1937.7 + 968.85}{6.0} = 484.42\ kN$$

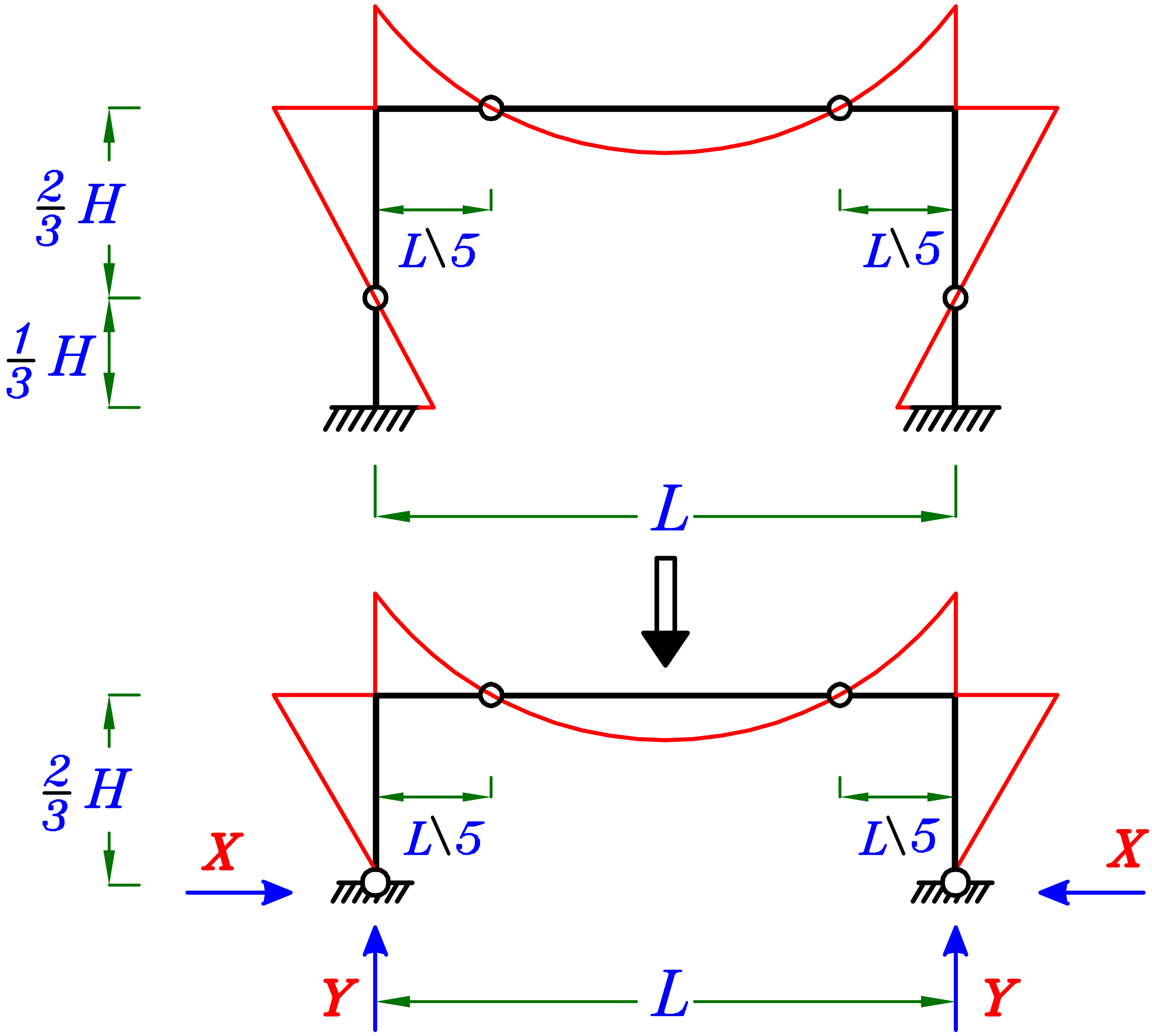
$$Y = \frac{\Sigma\ Load}{2} = \frac{62.62 * 21 + 2 * 130}{2} = 787.5\ kN$$


130
130
62.62 kN\m`
6.0
484.42 kN
484.42 kN
787.5 kN
787.5 kN
21.0
968.85 kN.m
968.85 kN.m


1937.7
1937.7
1937.7
1937.7
2617.95
B.M.D.
968.85
968.85


484.42
484.42
(–)
787.5
787.5
(–)
(–)
N.F.D.
787.5
787.5

Approximate Solution.



assume that in the column there is an intermediate hinge at $\frac{H}{3}$

so we can solve the Frame as Two hinged Frame but with height $\frac{2}{3}H$

assume that in the beam there is an intermediate hinge at $\frac{L}{5}$

$$Y = \frac{\sum Loads}{2}$$

To get the reactions X

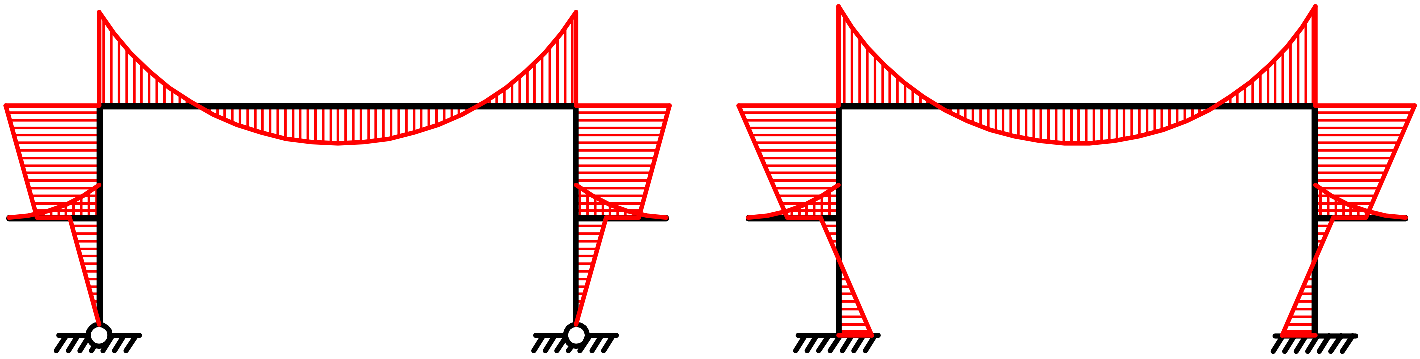
Take the moment at Point a = Zero

Then Draw Internal Forces Diagrams.

ملحوظه هامه

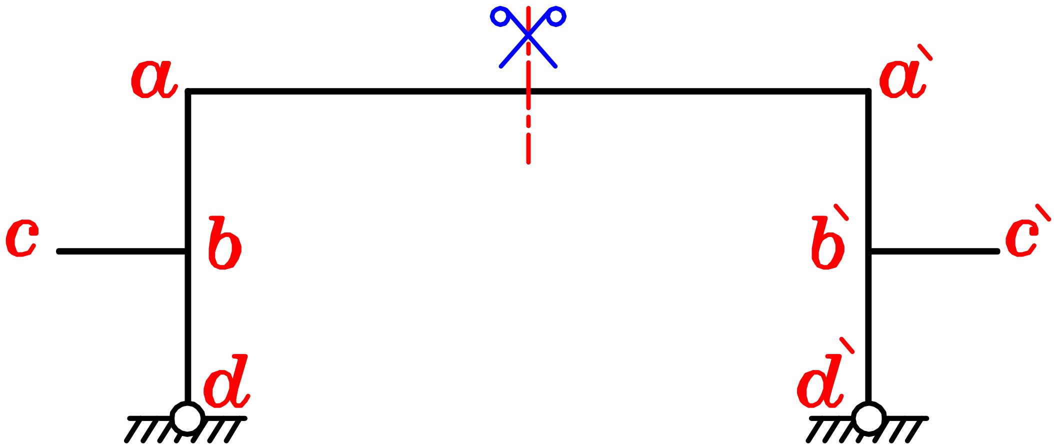
هذا الحل حل تقريبى جدا و غير دقيق ، لذا لن نستخدم هذا الحل الا مع تعذر الوقت فى الامتحان

Two Hinged or Fixed symmetric Frame with cantilever.



يتم حل ال **Frame** بطريقه ال **moment distribution**

و لكن يتعم عمل جدول لل **moment distribution**

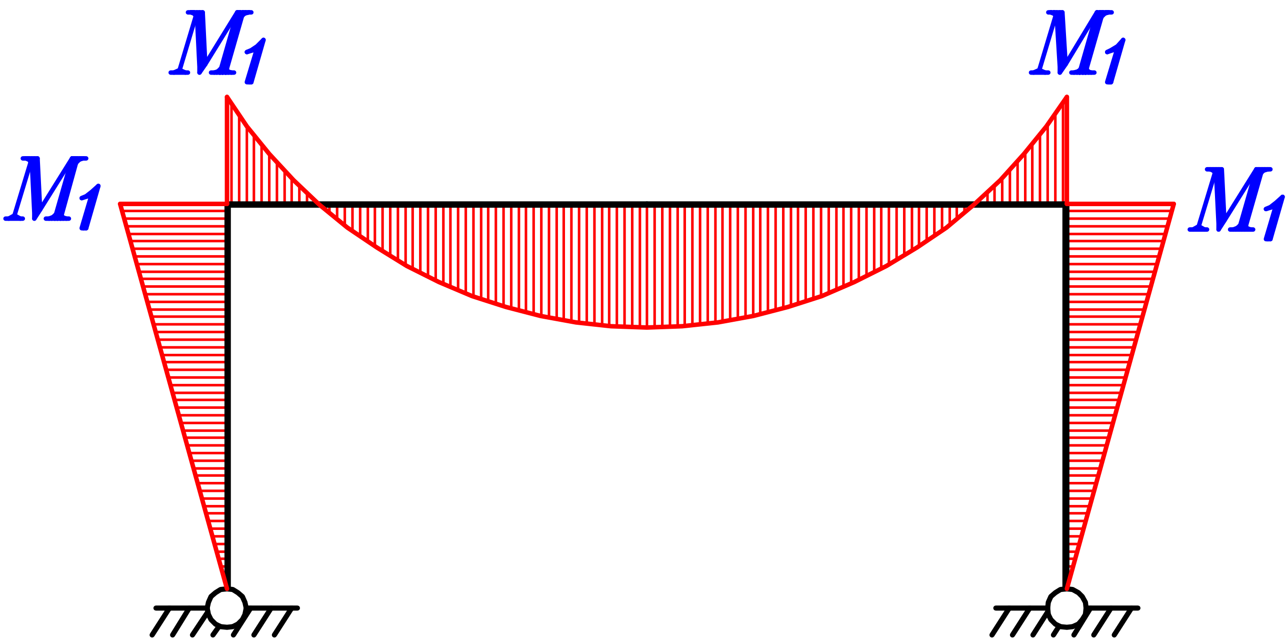


Joint	b			a	
member	b – d	b – c	b – a	a – b	a – a`
D.F.	✓	0	✓	✓	✓
F.E.M.	0	✓	0	0	✓
B.M.	✓	0	✓	✓	✓
C.O.M.	0	0	$\frac{1}{2}$ ✓	$\frac{1}{2}$ ✓	0
B.M.	✓	0	✓	✓	✓
C.O.M.	0	0	$\frac{1}{2}$ ✓	$\frac{1}{2}$ ✓	0
B.M.	✓	0	✓	✓	✓
M_F	✓	✓	✓	✓	✓

ممكن للتسهيل

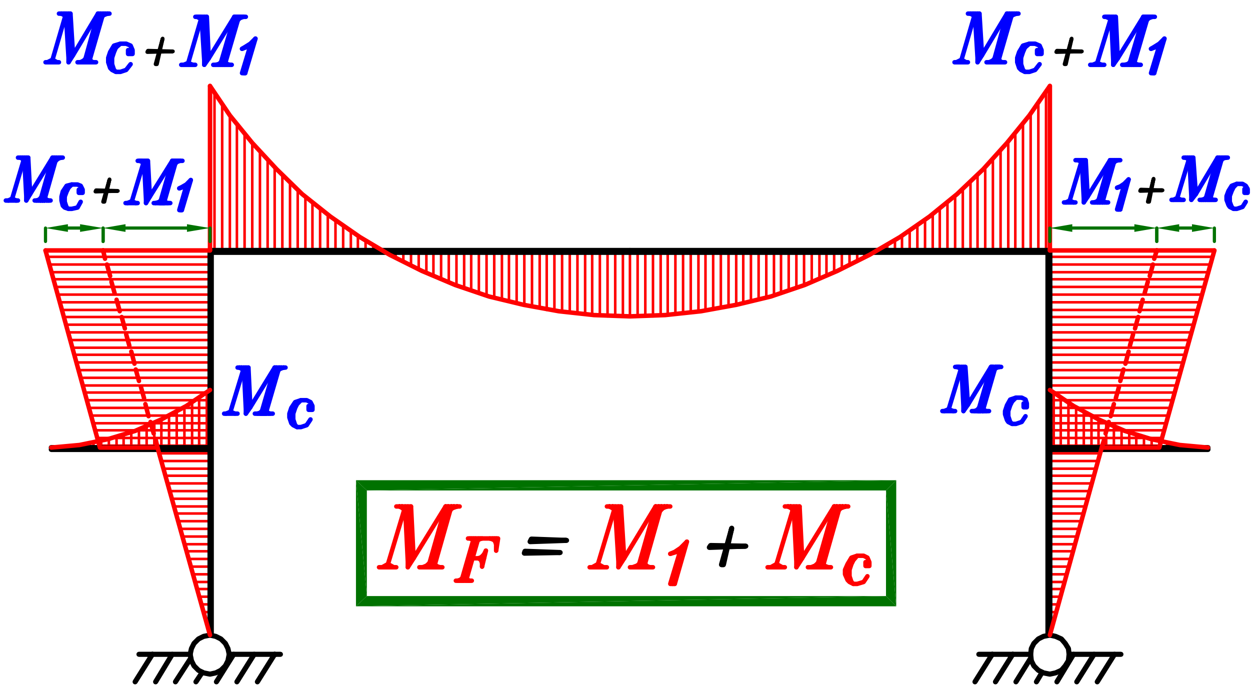
عند وجود cantilever خارج من ال symetric Frame

١- حل ال Frame بدون cantilever



$$\boxed{M_1 = F.E.M._{(beam)} * D.F._{(col.)}}$$

٢- حساب قيمة العزم النهائى $M_F = M_1 + M_c$

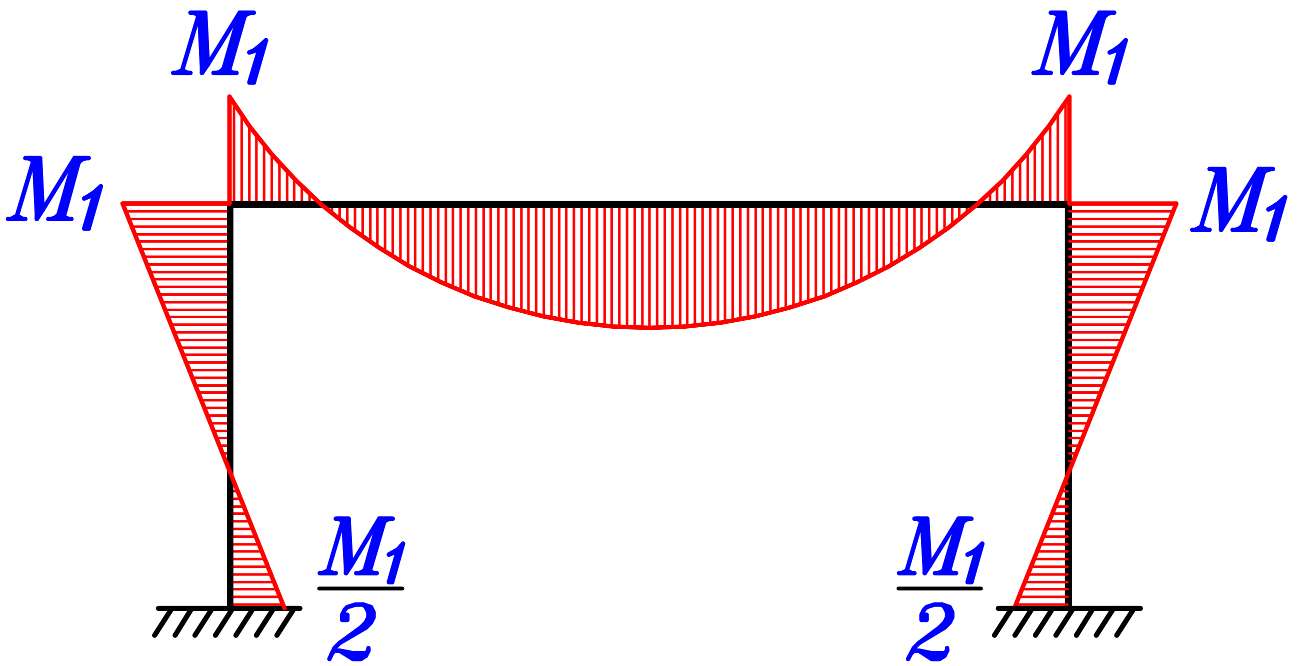


$$\boxed{M_F = M_1 + M_c}$$

ممكن للتسهيل

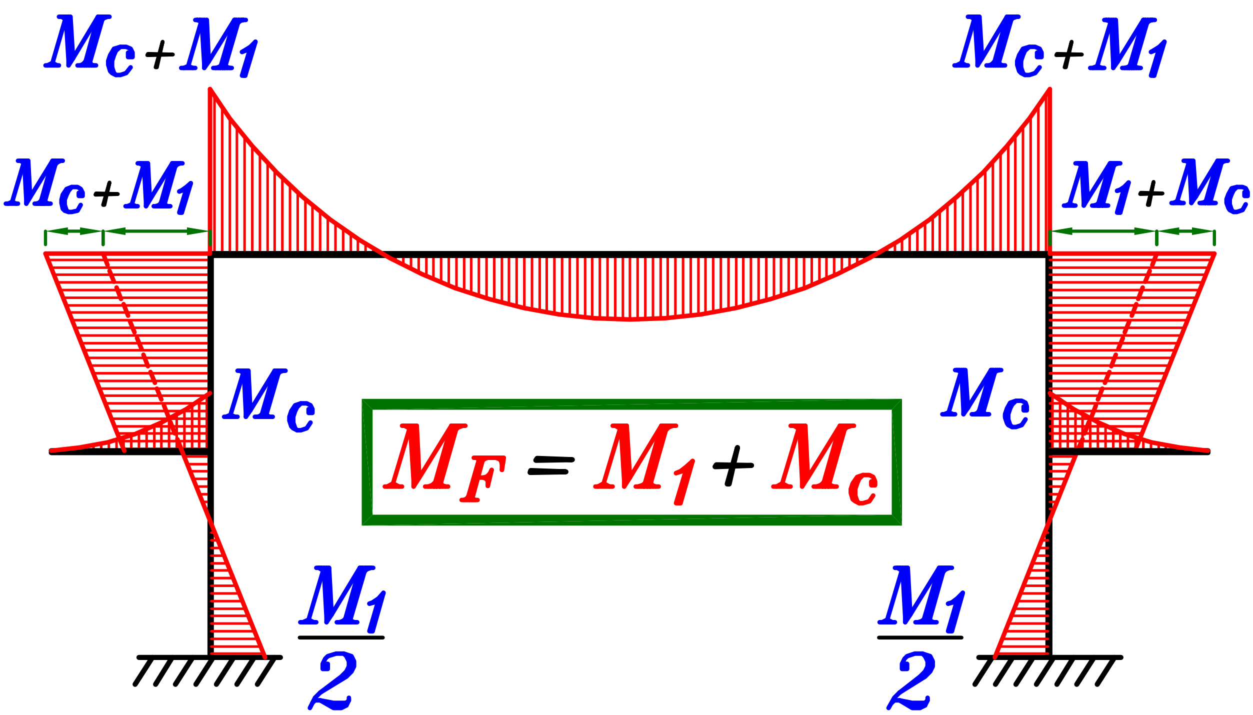
عند وجود cantilever خارج من ال symetric Frame

١- حل ال Frame بدون cantilever



$$\boxed{M_1 = F.E.M._{(beam)} * D.F._{(col.)}}$$

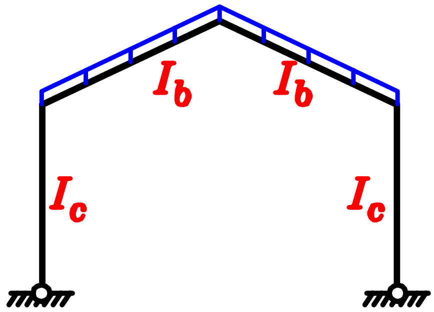
٢- حساب قيمه العزم النهائى $M_F = M_1 + M_c$



② Virtual Work Method.

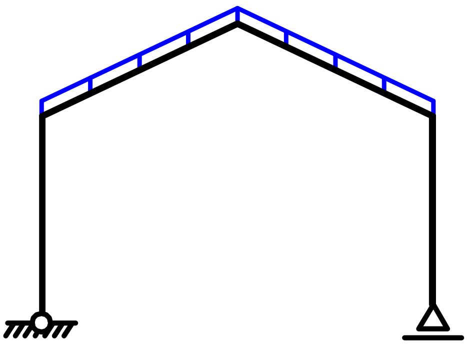
في طريقه **Virtual Work** للتسهيل يتم تحويل الاحمال المركزه **Concentrated loads**
الى احمال منتظمه **Distibuted loads** حتى لو كان عدد **Concentrated loads** أقل من ٣

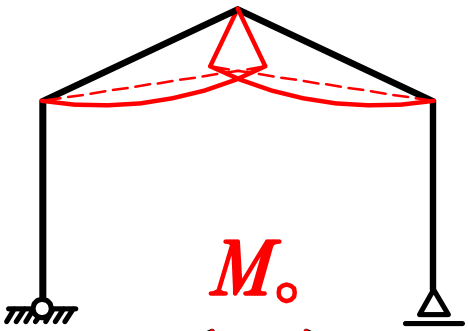
① IF there is a Sway.

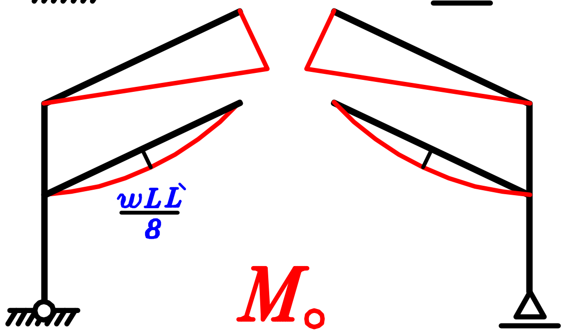


ⓐ Get Moment of Inertia For all members. I

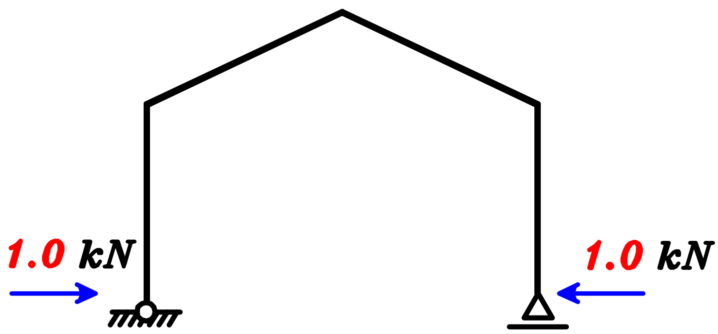
ⓑ Make the Frame Determinate and Draw M_o

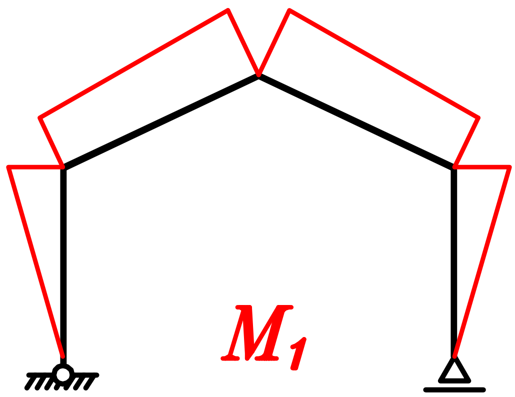






ⓒ Draw M_1 نحذف كل الاحمال و نضع 1.0 kN فى اتجاه المجهول





ⓓ Calculate the deflections δ_{10} , δ_{11}

$$\delta_{10} = \frac{1}{E_c I_b} * (M_o * M_1) + \frac{1}{E_c I_c} * (M_o * M_1)$$

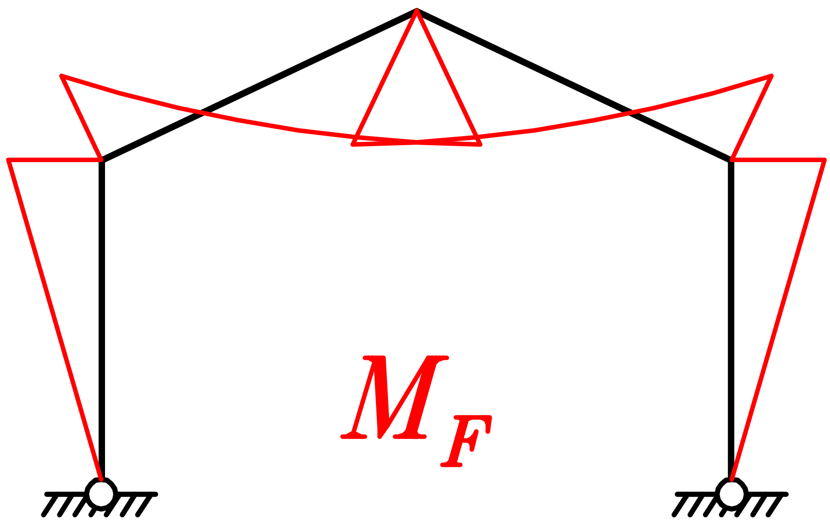
$$\delta_{11} = \frac{1}{E_c I_b} * (M_1 * M_1) + \frac{1}{E_c I_c} * (M_1 * M_1)$$

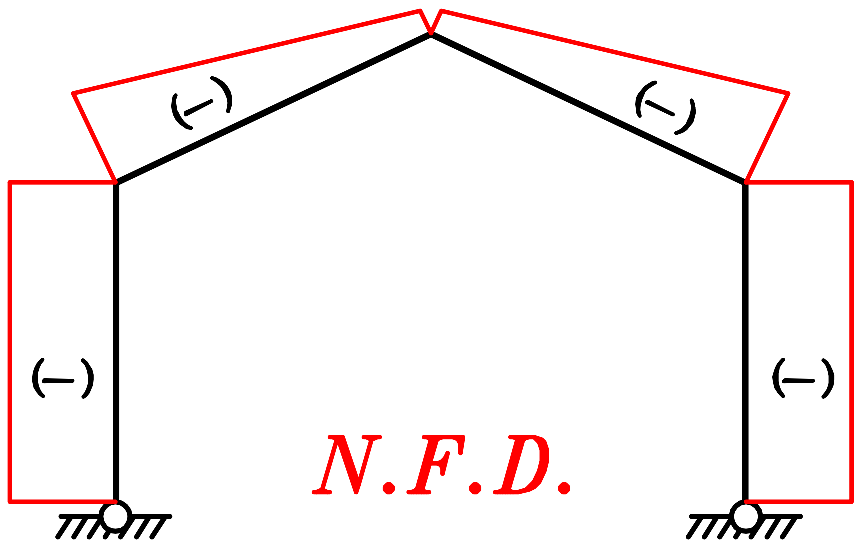
ⓔ Calculate X

$$\boxed{\delta_{10} + X \; \delta_{11} = Zero}$$ Get X

ⓕ Calculate M_F

$$\boxed{M_F = M_o + X \; M_1}$$

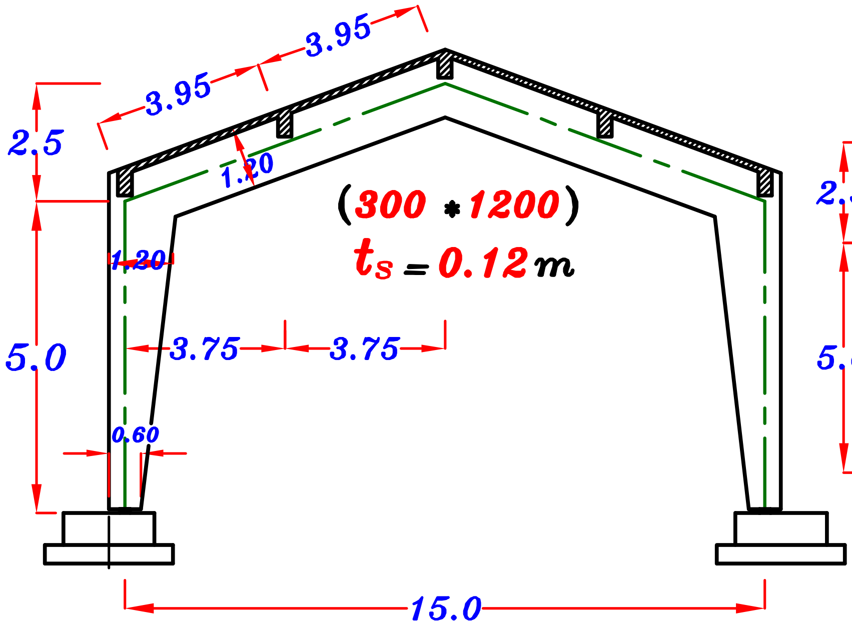


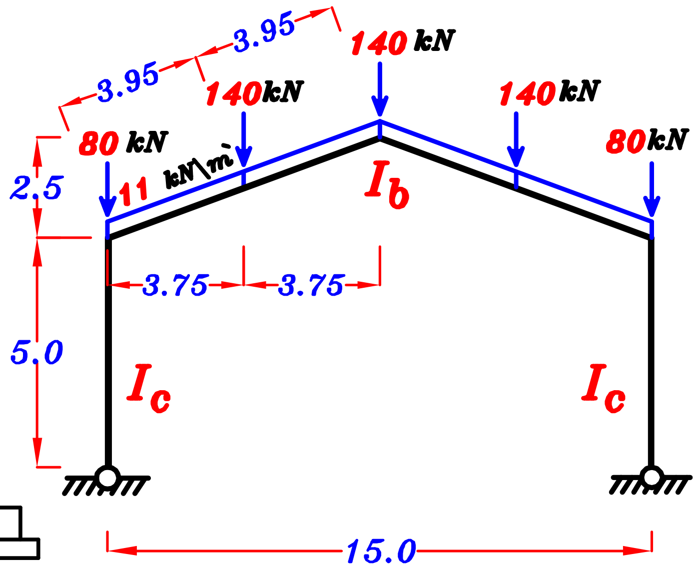


Two Hinged Inclined Frame.

Example.

For the given Frame, Draw B.M.D. & N.F.D.





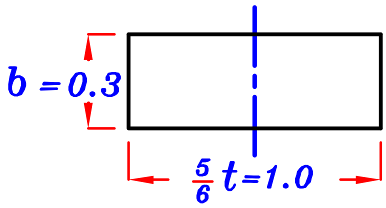
For the Two hinged Inclined Frame we will use Virtual Work Method.

Solution.

I_c

$$I_c = \frac{b\left(\frac{5}{6}t\right)^3}{12} = \frac{0.3\left(\frac{5}{6} * 1.20\right)^3}{12}$$

$$= 0.025\ m^4$$



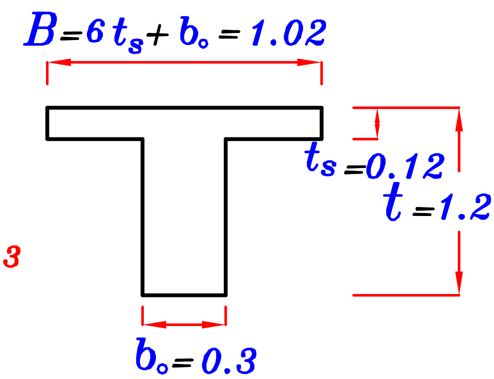
I_b

$$\frac{t_s}{t} = \frac{0.12}{1.20} = 0.10$$

$$\frac{b_\circ}{B} = \frac{0.3}{1.02} = 0.294$$

Table Page 63

$$\mu = 360$$



$$I_b = (\mu * 10^{-4})\,B\,t^3 = 360 * 10^{-4} * 1.02 * 1.2^3$$

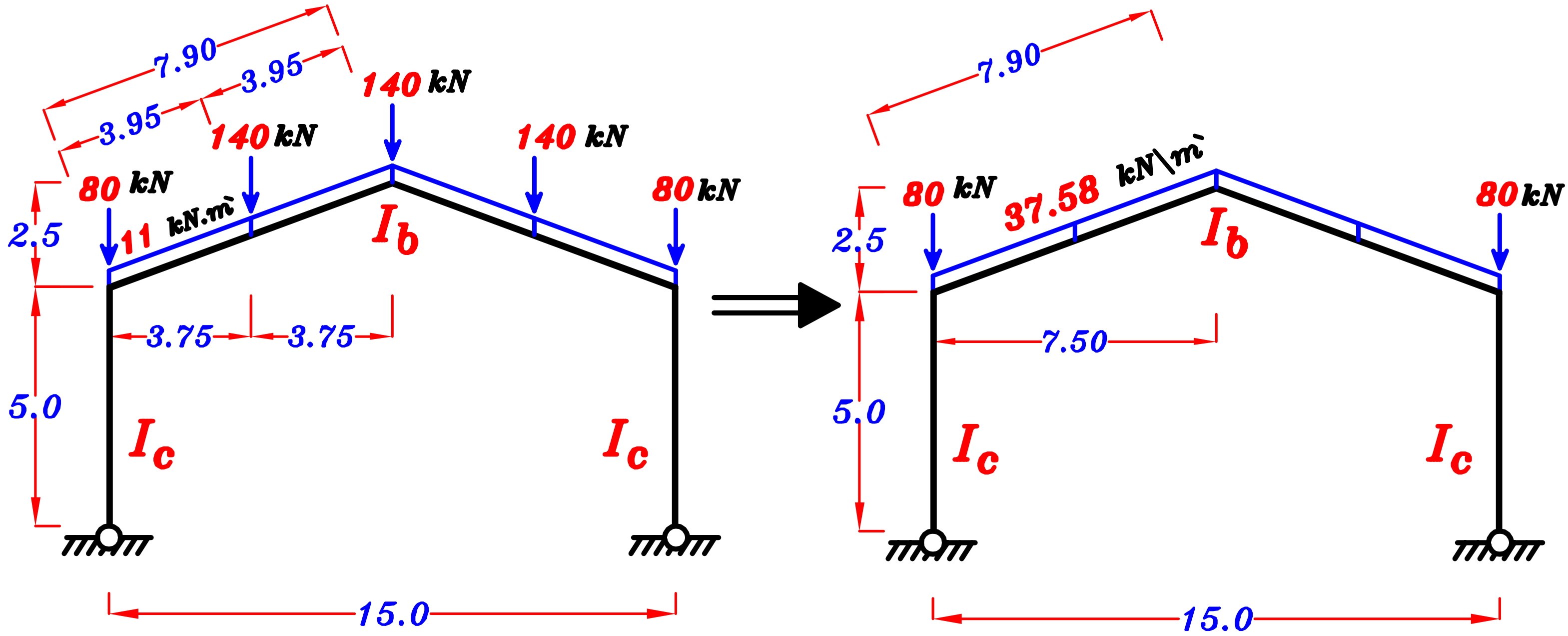
$$= 0.0634\ m^4$$

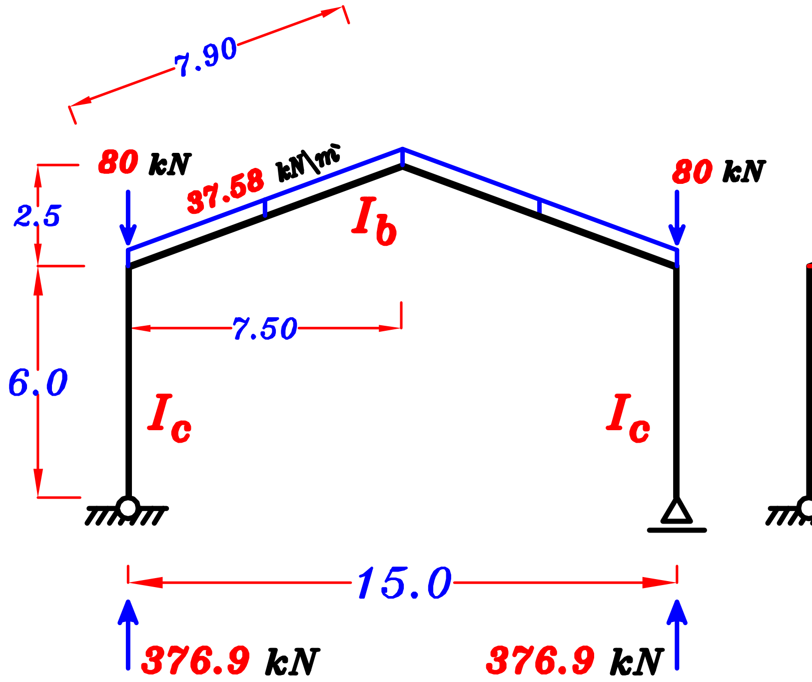
$$\therefore \boxed{I_b = 2.54\ I_c}$$

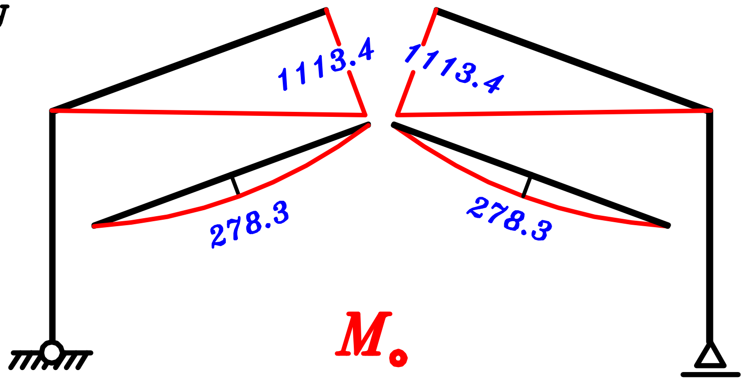
للتسهيل يتم تحويل الاحمال المركزه **Concentrated loads**

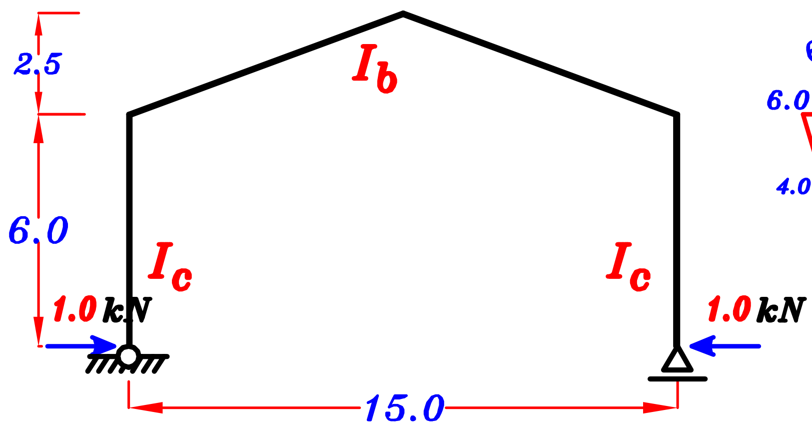
الى احمال منتظمه **Distibuted loads**

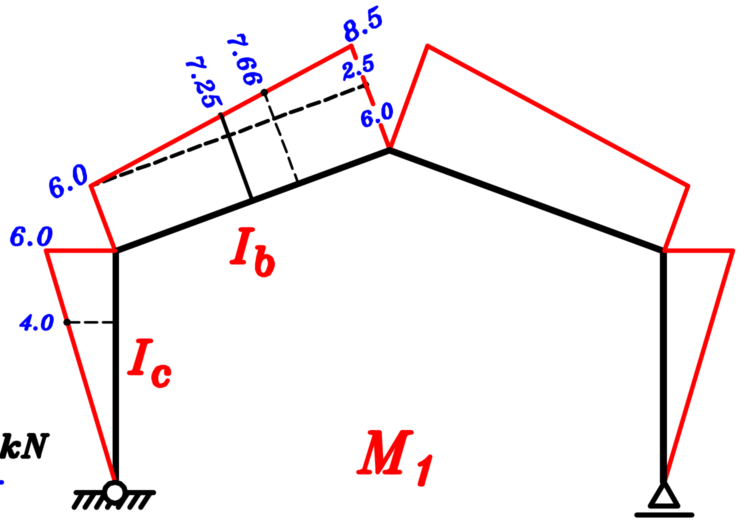
$$w = o.w. + \frac{\sum P}{span} = 11.0 + \frac{3\,(140)}{2 * 7.9} = 37.58 \ \ kN\backslash m`$$











$$\delta_{1o} = \frac{1}{E_c I_c} * (M_o * M_1) + \frac{1}{E_c I_b} * (M_o * M_1)$$

$$\delta_{1o} = zero + \frac{-2}{E_c (2.54) I_c} \left(\tfrac{1}{2}(7.90)(1113.4)[7.66] + \tfrac{2}{3}(278.3)(7.90)[7.25] \right) = \frac{-34893.35}{E_c I_c}$$

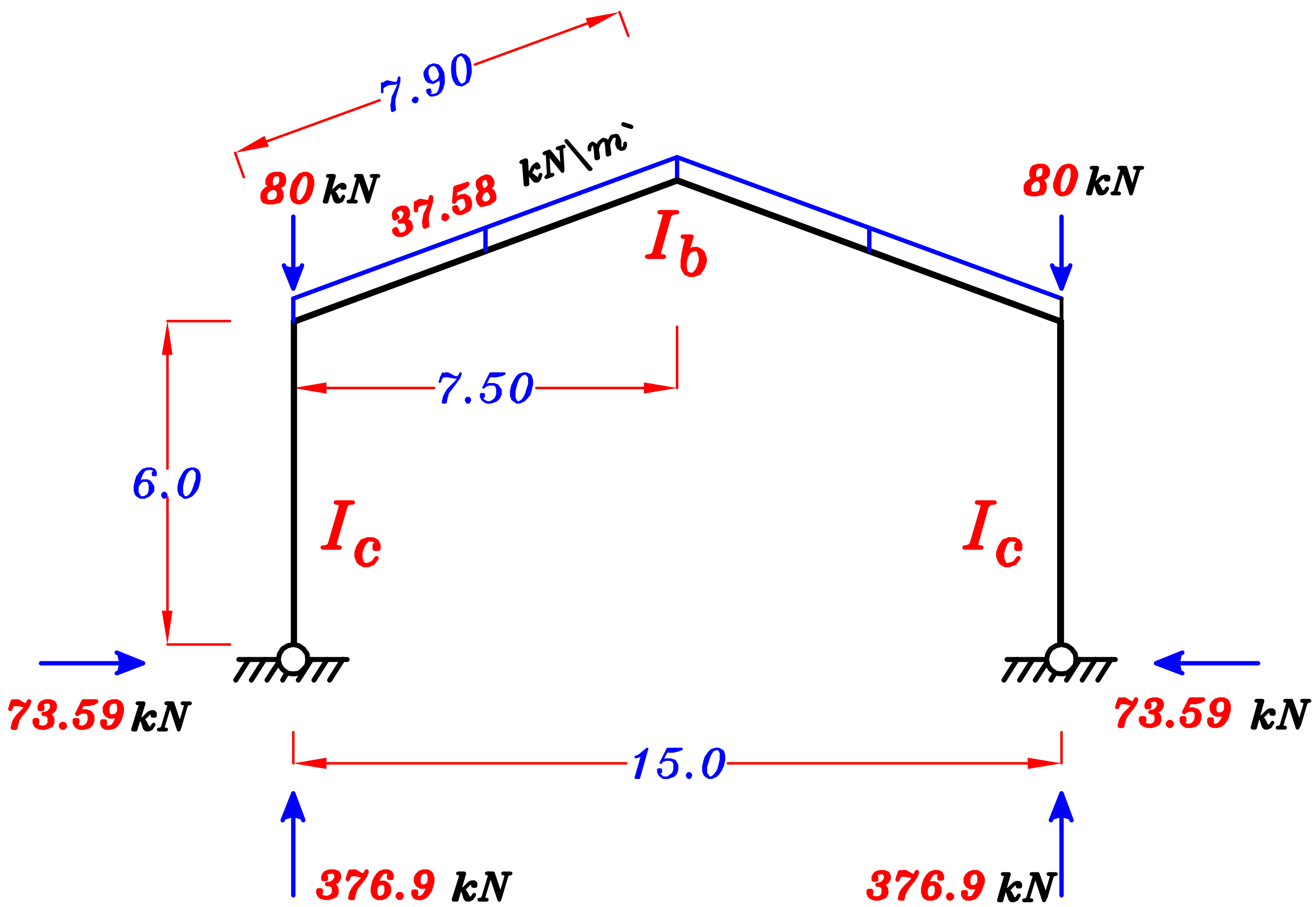
$$\delta_{11} = \frac{1}{E_c I_c} * (M_1 * M_1) + \frac{1}{E_c I_b} * (M_1 * M_1)$$

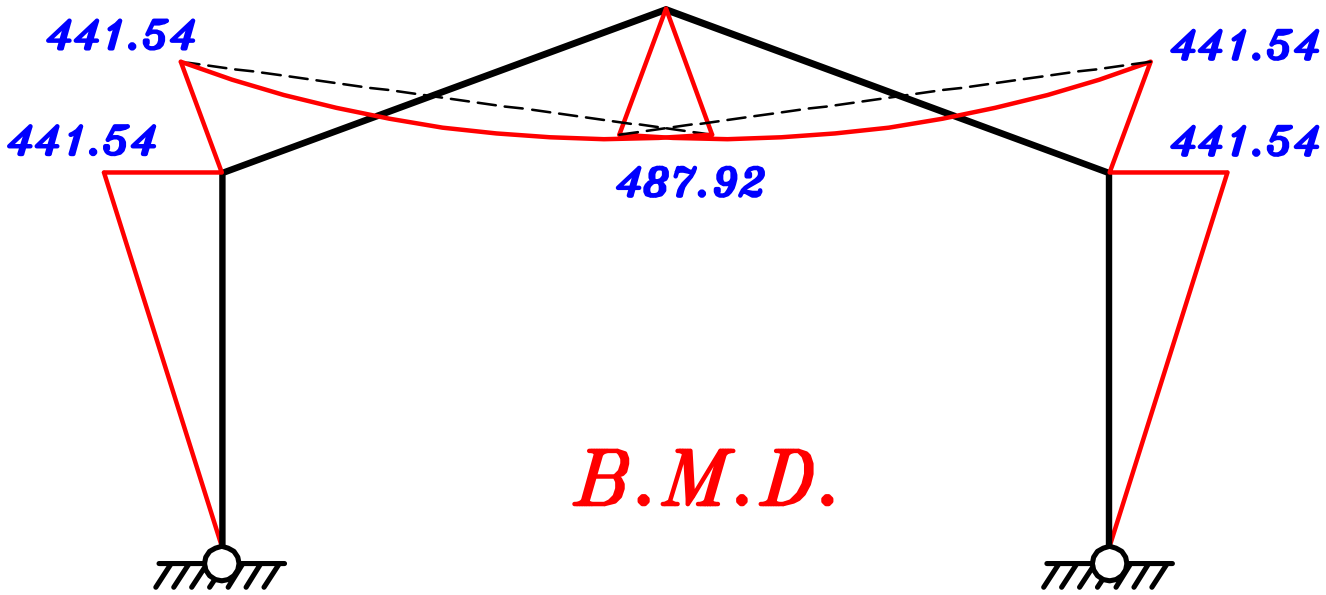
$$\delta_{11} = \frac{2}{E_c I_c} \left(\tfrac{1}{2}(6)(6)[4.0] \right) + \frac{2}{E_c (2.54) I_c} \left((6)(7.9)[7.25] + \tfrac{1}{2}(7.9)(2.5)[7.66] \right)$$

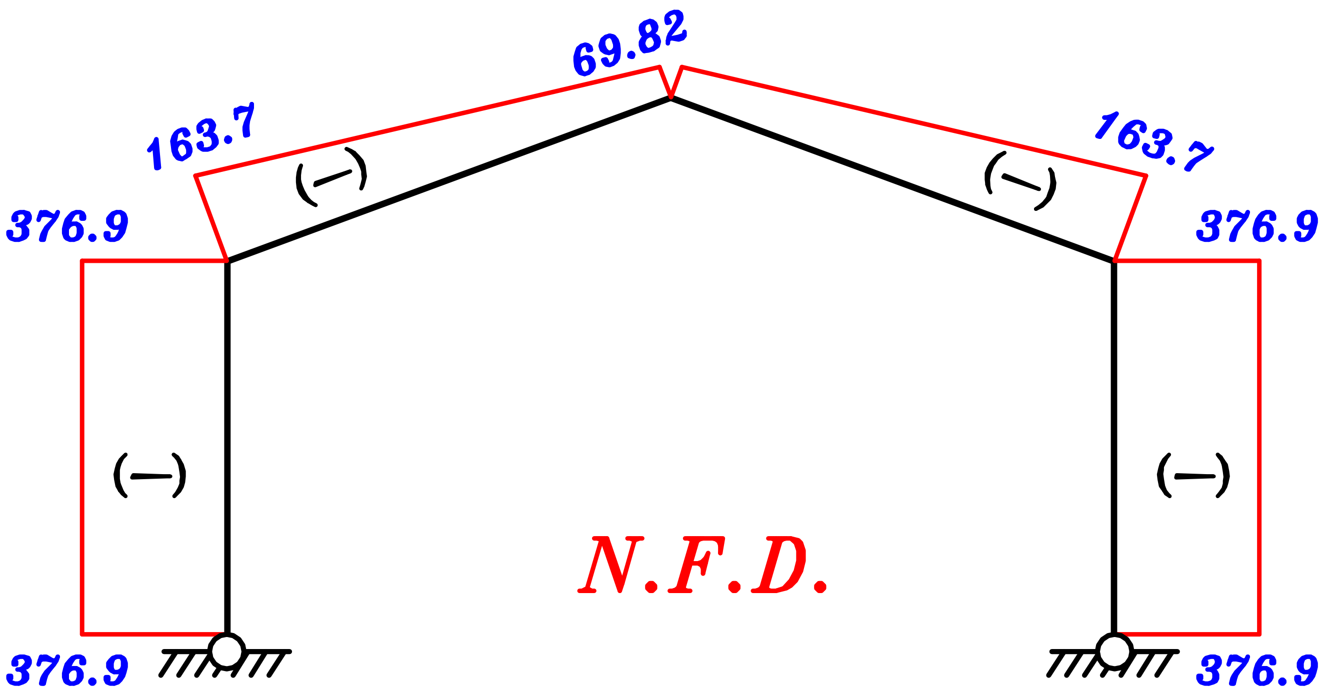
$$= \frac{144}{E_c I_c} + \frac{330.15}{E_c I_c} = \frac{474.15}{E_c I_c}$$

$$\because \quad \delta_{1o} + X\,\delta_{11} = Zero = \frac{-34893.35}{E_c I_c} + X * \frac{474.15}{E_c I_c}$$

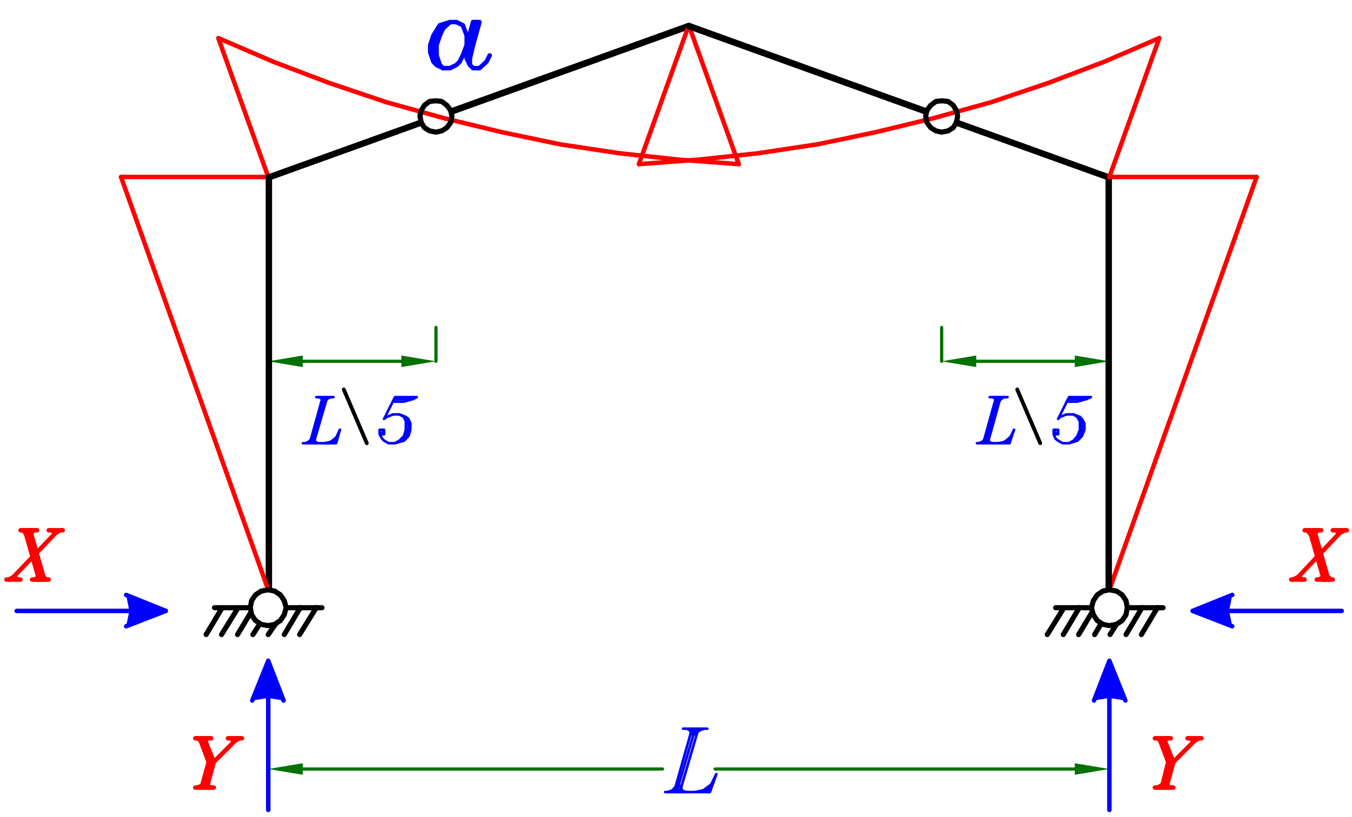
$$\therefore \quad \boxed{X = 73.59\ kN}$$




441.54
441.54
441.54
441.54
487.92
B.M.D.


69.82
163.7
163.7
(−)
(−)
376.9
376.9
(−)
(−)
N.F.D.
376.9
376.9

Approximate Solution.



assume that in the beam there is an intermediate hinge at $\frac{L}{5}$

$$Y = \frac{\sum Loads}{2}$$

To get the reactions X

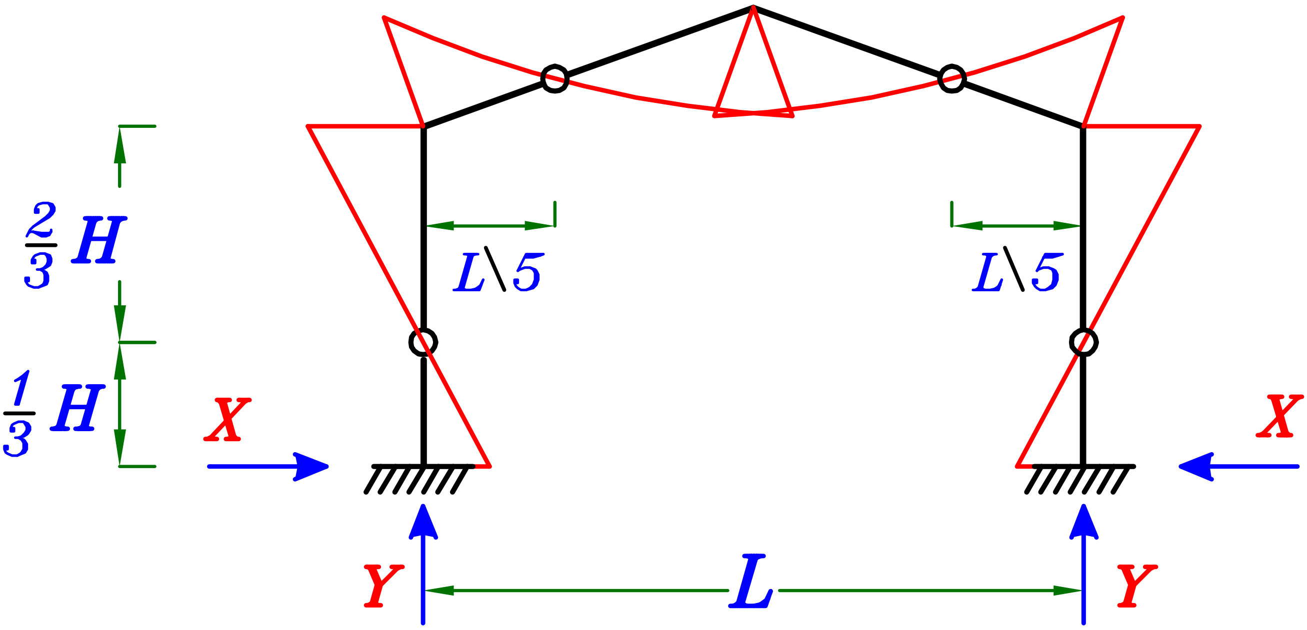
Take the moment at Point a = Zero

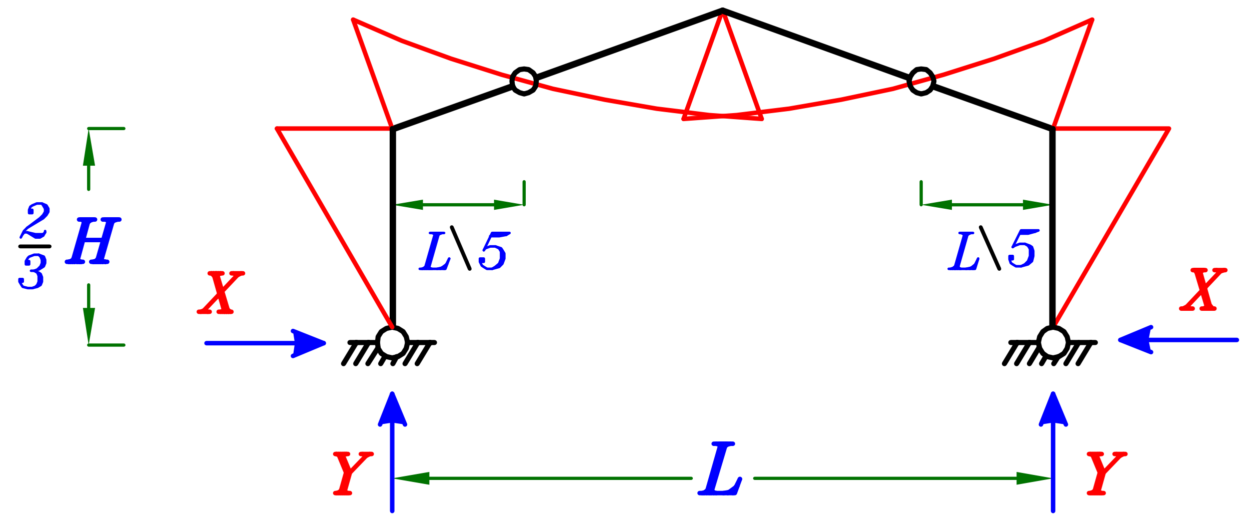
Then Draw Internal Forces Diagrams.

ملحوظه هامه

هذا الحل حل تقريبى جدا و غير دقيق ، لذا لن نستخدم هذا الحل الا مع تعذر الوقت فى الامتحان .

Inclined Fixed Frames.





assume that in the column there is an intermediate hinge at $\frac{H}{3}$

so we can solve the Frame as Two hinged Frame but with height $\frac{2}{3}H$

assume that in the beam there is an intermediate hinge at $\frac{L}{5}$

$$Y = \frac{\sum Loads}{2}$$

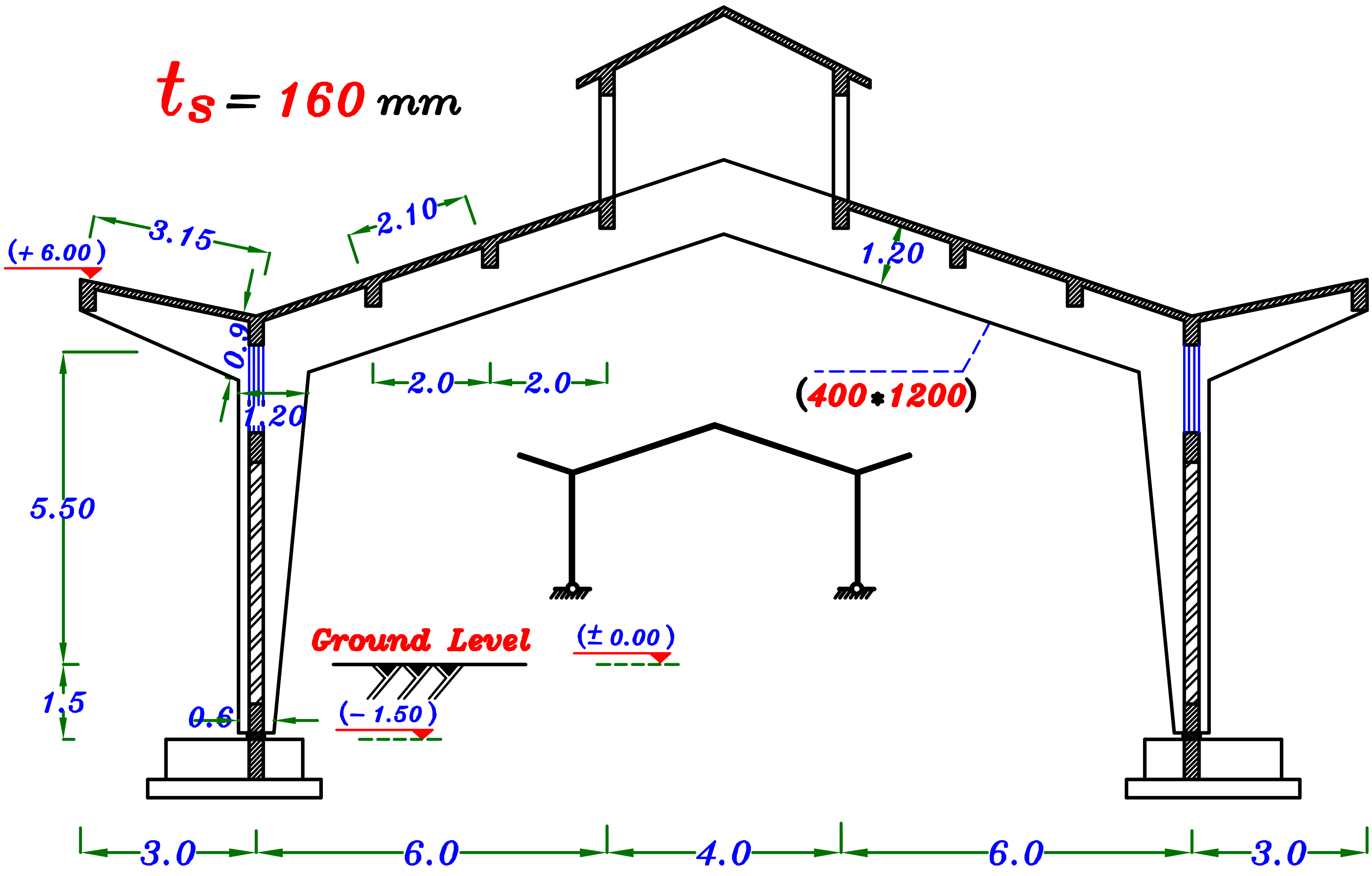
To get the reactions X

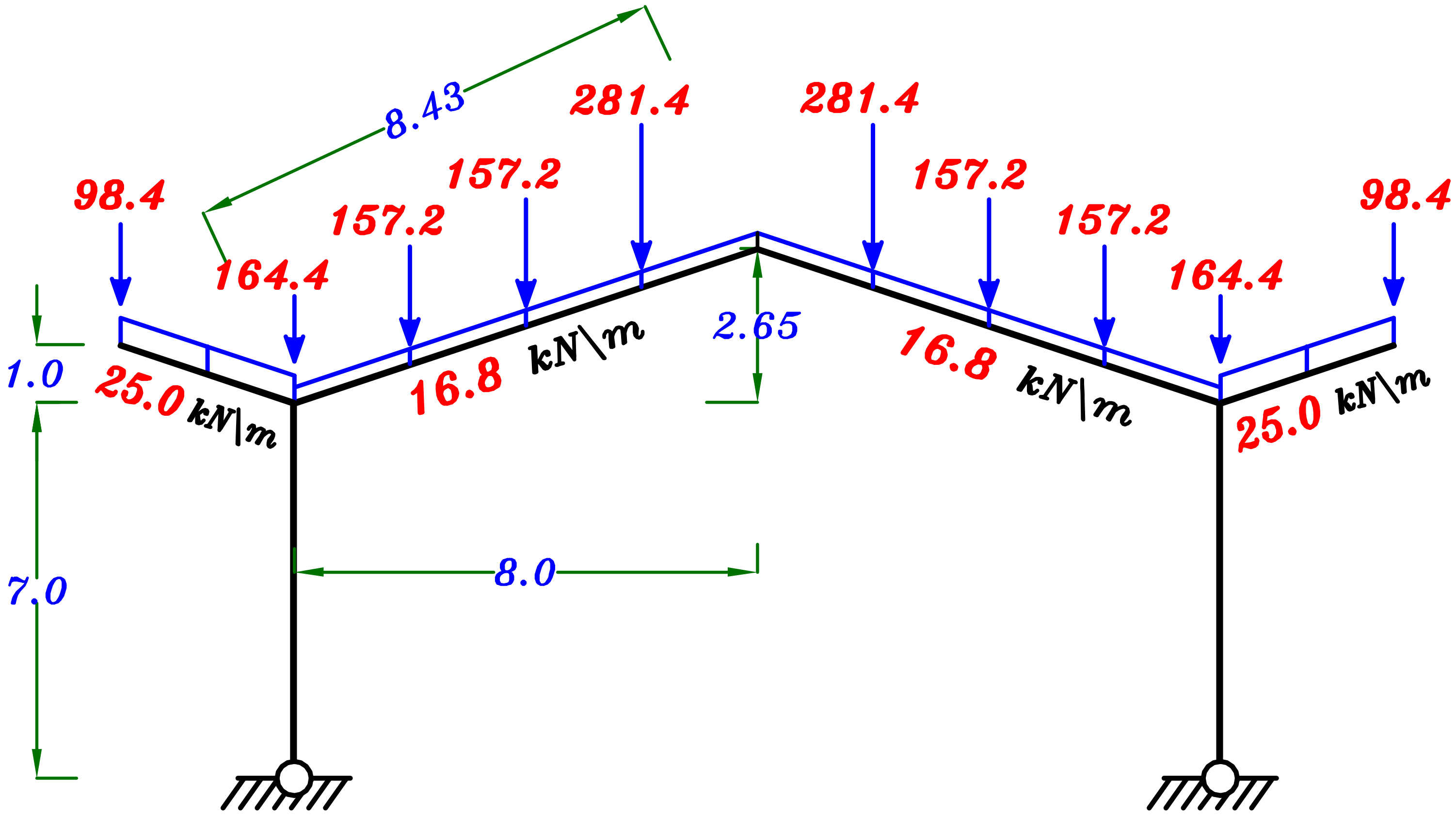
Take the moment at Point a = Zero

Example.

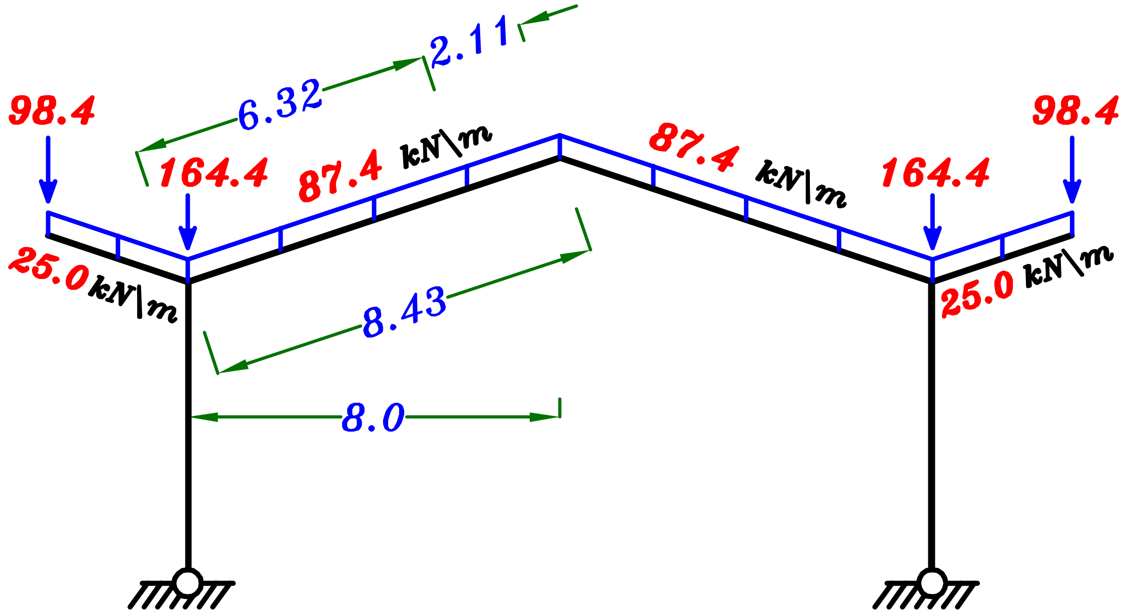
Draw B.M.D. , N.F.D For the given Frame.

$t_s = 160$ mm





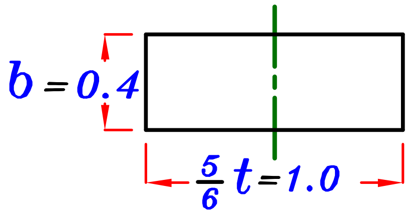
$$w = o.w. + \frac{\sum P}{span} = 16.8 + \frac{2(281.4) + 4(157.2)}{2(8.43)} = 87.4 \ kN\backslash m$$



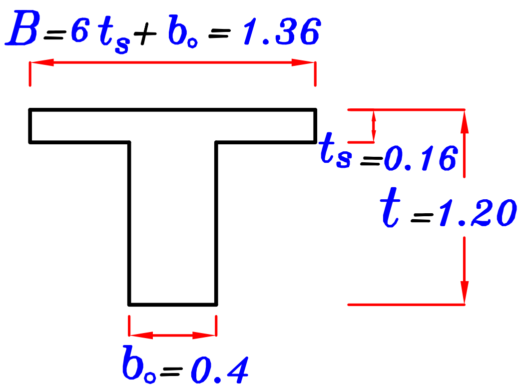
I_c

$$I_c = \frac{b\left(\frac{5}{6}t\right)^3}{12} = \frac{0.4\left(\frac{5}{6} * 1.20\right)^3}{12}$$

$$= 0.033333 \ m^4$$



I_{b1}



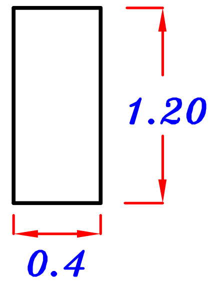
$$\left.\begin{aligned} \frac{t_s}{t} &= \frac{0.16}{1.20} = 0.134 \\ \frac{b_o}{B} &= \frac{0.4}{1.36} = 0.294 \end{aligned}\right\}$$ **Table Page 63** $\mu = 382$

$$I_b = (\mu * 10^{-4})\, B t^3 = 382 * 10^{-4} * 1.36 * 1.20^3$$

$$= 0.0897 \ m^4$$

I_{b2}

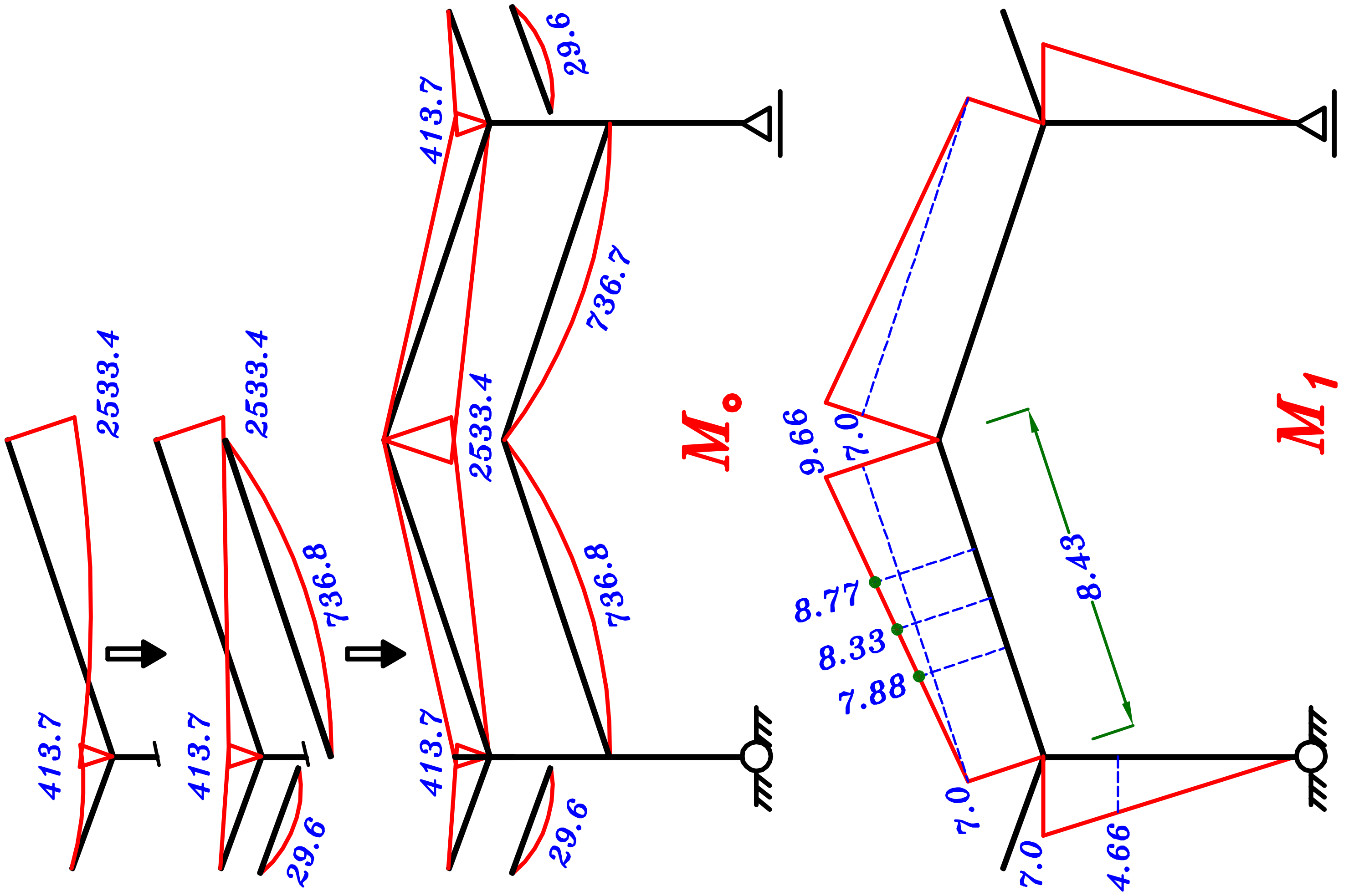
$$I_{b2} = \frac{b\,(t)^3}{12} = \frac{0.4\,(1.20)^3}{12} = 0.0576 \ m^4$$



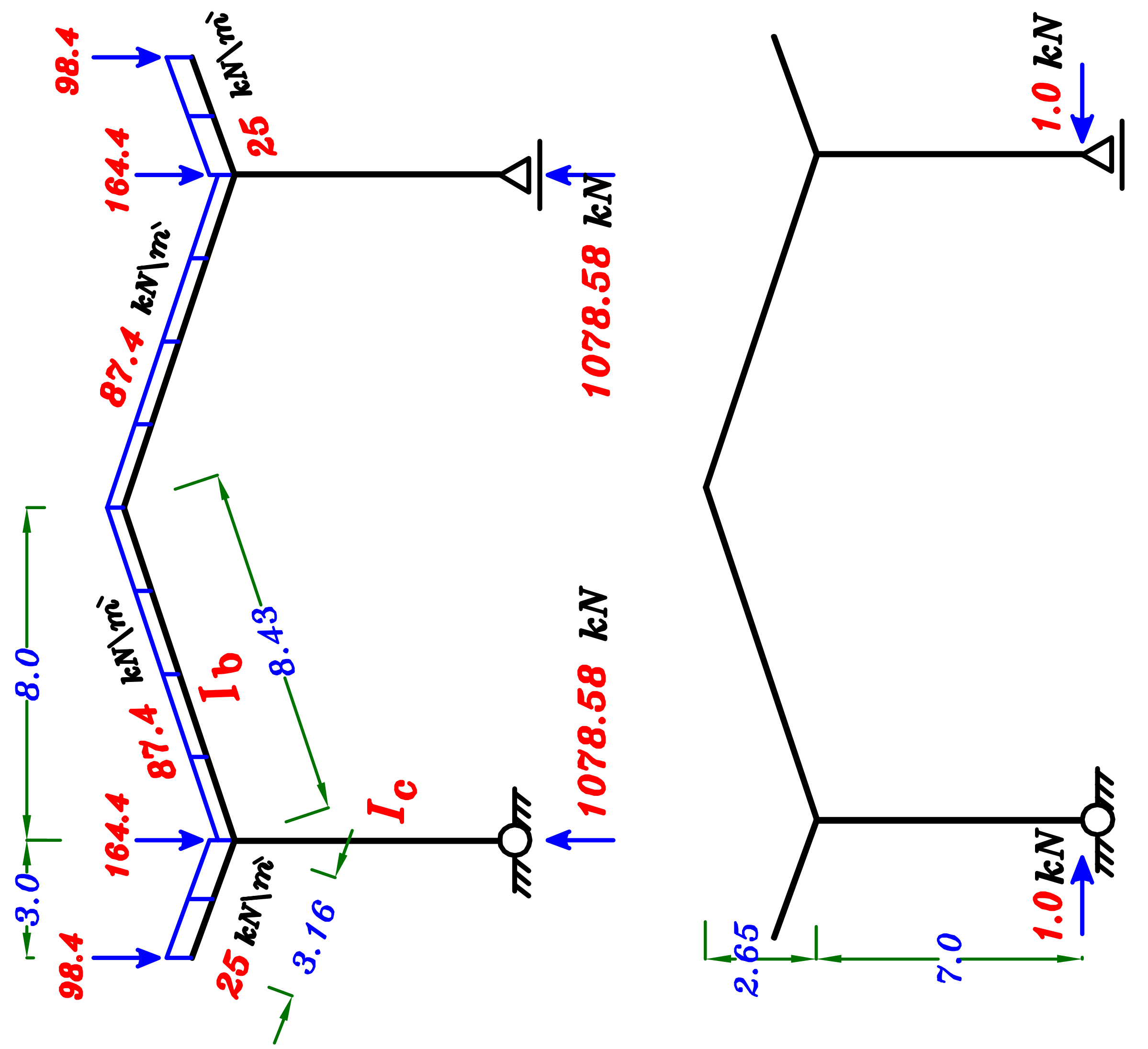
I_b نأخذ المتوسط

$$I_b = \frac{0.0897 * 2 * 6.32 + 0.0576 * 2 * 2.11}{2 * 8.43} = 0.08167$$

$$\therefore \quad \boxed{I_b = 2.45\, I_c}$$



413.7
2533.4
736.8
29.6
736.7
M_o
M_1
9.66
7.0
8.77
8.33
7.88
8.43
4.66



98.4
164.4
25 kN\m`
87.4 kN\m`
1078.58 kN
I_b
I_c
8.0
3.0
8.43
3.16
1.0 kN
2.65
7.0

$$\delta_{10} = \frac{1}{E_c I_c} * (M_o * M_1) + \frac{1}{E_c I_b} * (M_o * M_1)$$

$$\delta_{10} = zero + \frac{2}{E_c (2.45) I_c} \Big(-\tfrac{1}{2}(8.43)(2533.4)[8.77] + \tfrac{1}{2}(8.43)(413.7)[7.88] - \tfrac{2}{3}(736.8)(8.43)[8.33] \Big) = \frac{-93388.41}{E_c I_c}$$

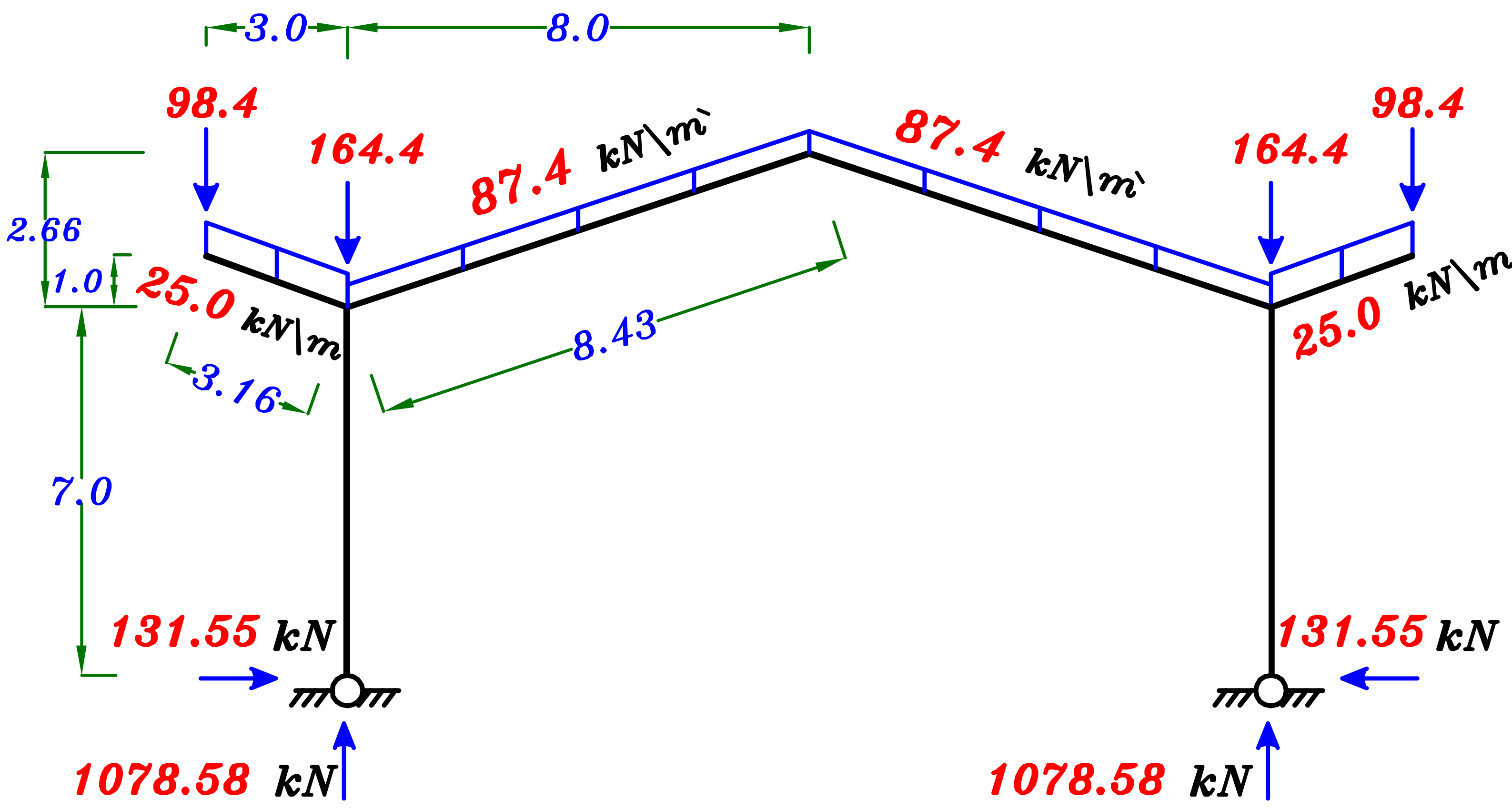
$$\delta_{11} = \frac{1}{E_c I_c} * (M_1 * M_1) + \frac{1}{E_c I_b} * (M_1 * M_1)$$

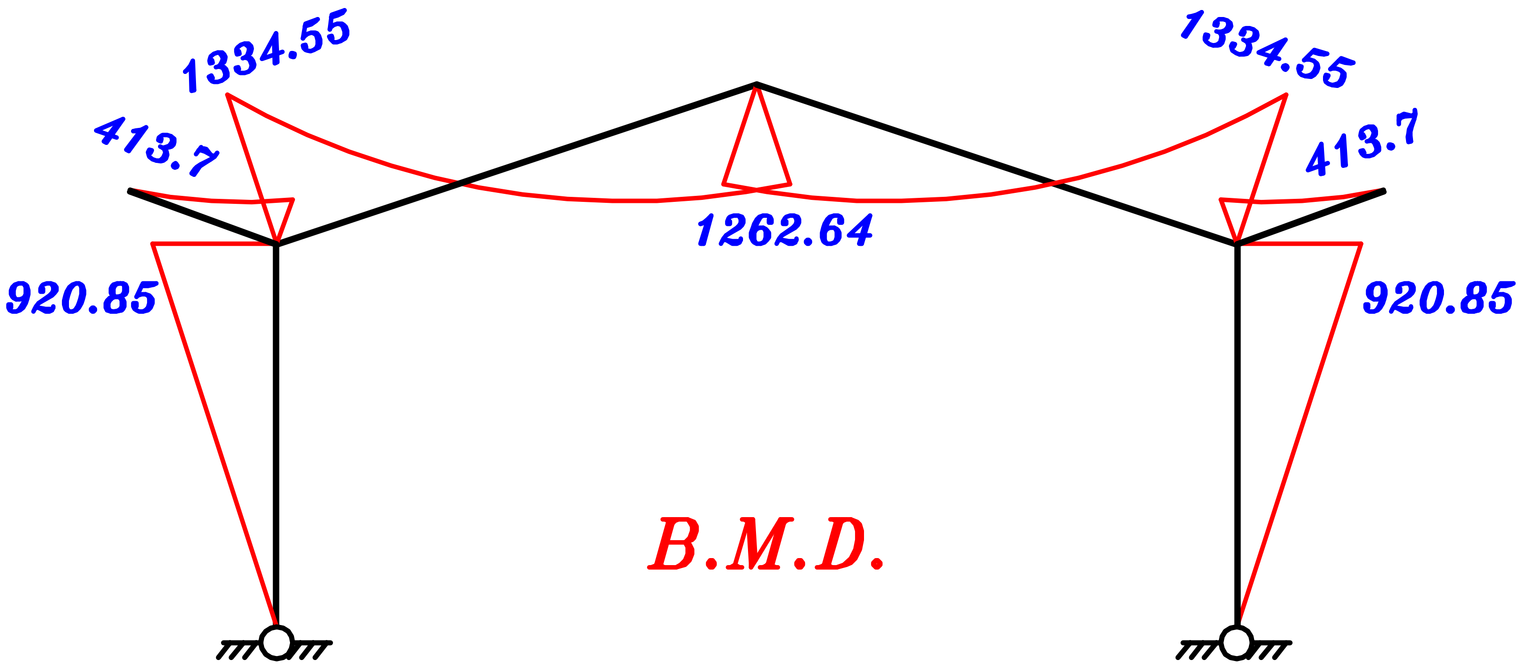
$$\delta_{11} = \frac{2}{E_c I_c} \Big(\tfrac{1}{2}(7.0)(7.0)[4.66] \Big) + \frac{2}{E_c (2.45) I_c} \Big((7.0)(8.43)[8.33] + \tfrac{1}{2}(8.43)(2.66)[8.77] \Big)$$

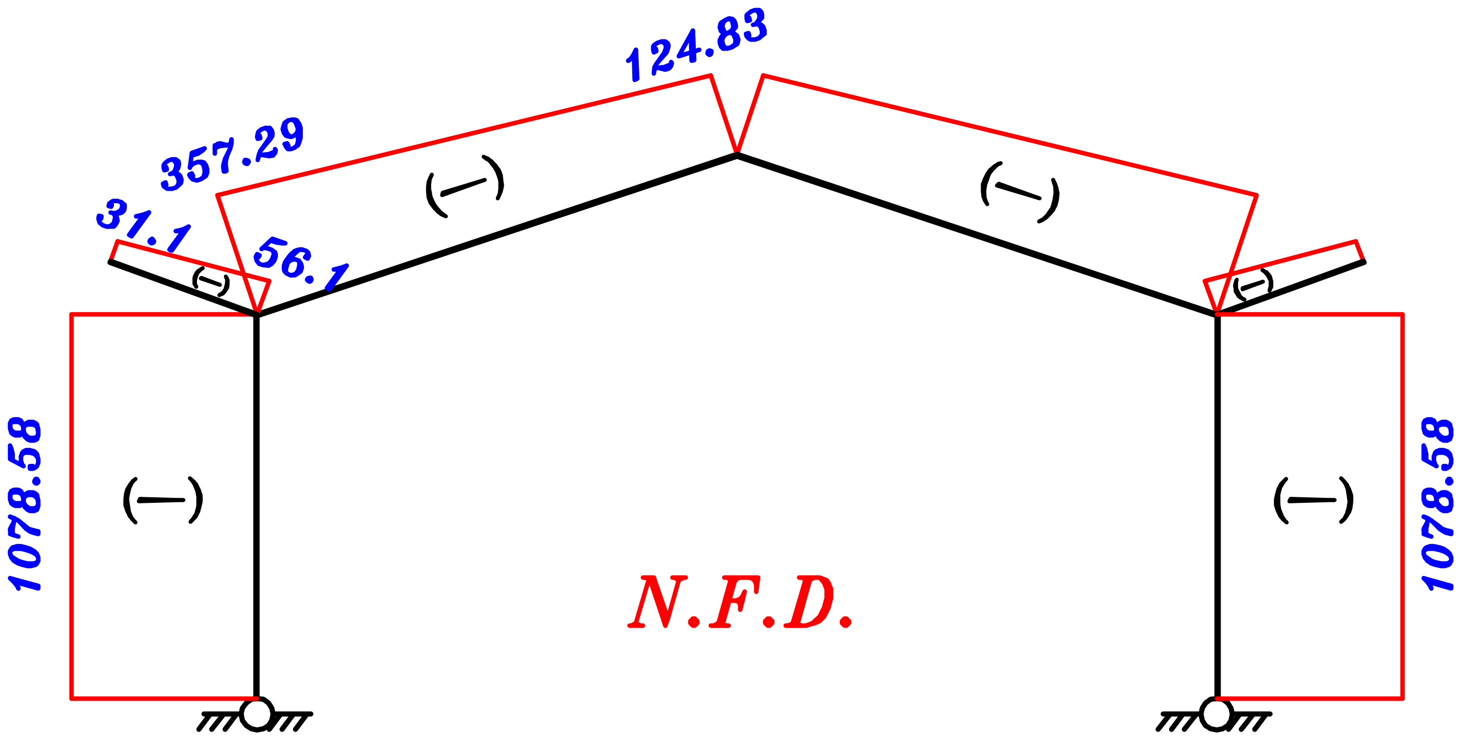
$$= \frac{228.34}{E_c I_c} + \frac{481.53}{E_c I_c} = \frac{709.87}{E_c I_c}$$

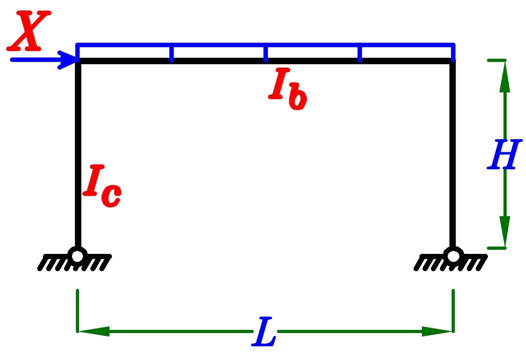
$$\because \quad \delta_{10} + X \, \delta_{11} = Zero$$

$$= \frac{-93388.41}{E_c I_c} + X * \frac{709.87}{E_c I_c} \longrightarrow \boxed{X = 131.55 \text{ kN}}$$



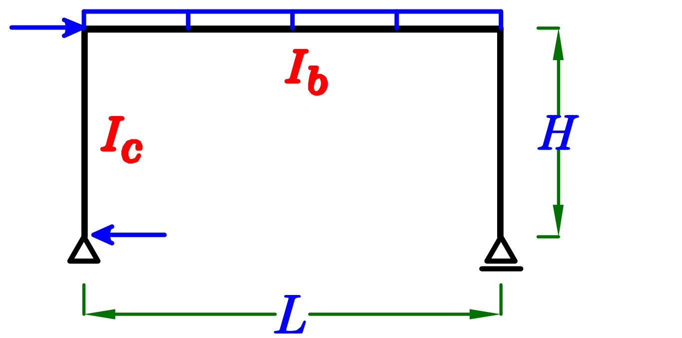

1334.55
1334.55
413.7
413.7
1262.64
920.85
920.85
B.M.D.

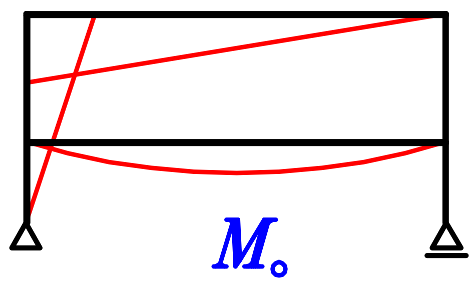

124.83
357.29
31.1
56.1
(—)
(—)
(—)
(—)
1078.58
1078.58
(—)
(—)
N.F.D.

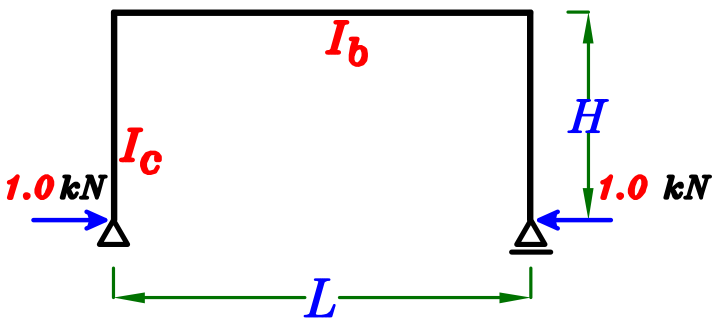


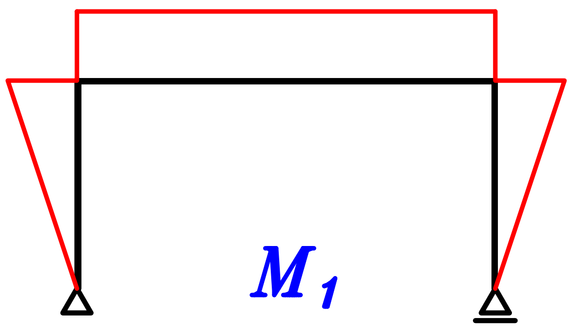
∵ There is a sway on the Frame

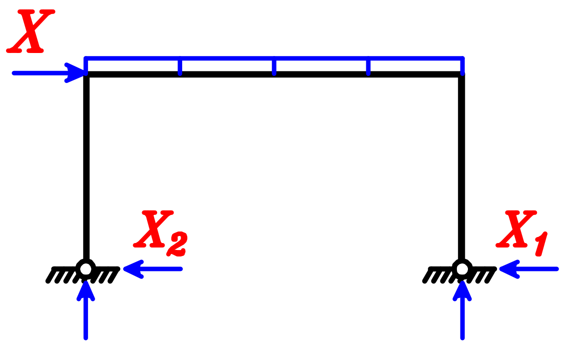
∴ We will solve using Virtual Work Method.







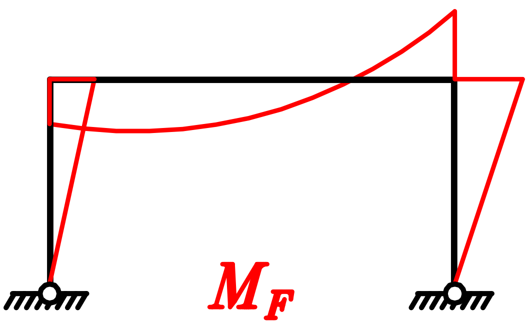




$$\delta_{1o} = \frac{1}{E_c I_b} * (M_o * M_1) + \frac{1}{E_c I_c} * (M_o * M_1)$$

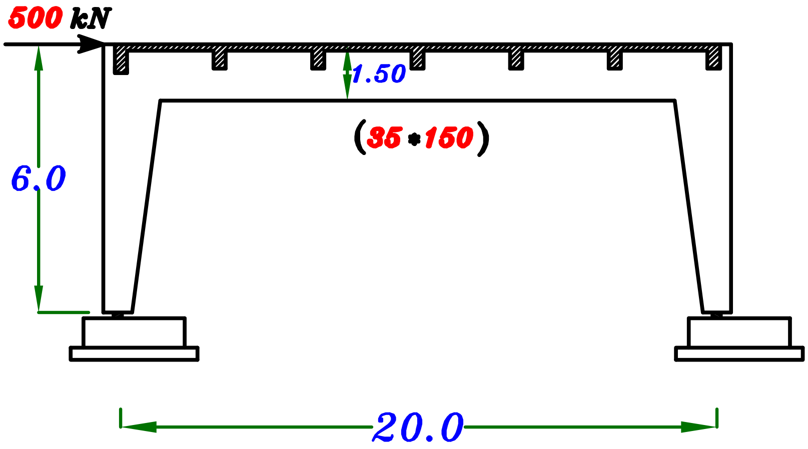
$$\delta_{11} = \frac{1}{E_c I_b} * (M_1 * M_1) + \frac{1}{E_c I_c} * (M_1 * M_1)$$

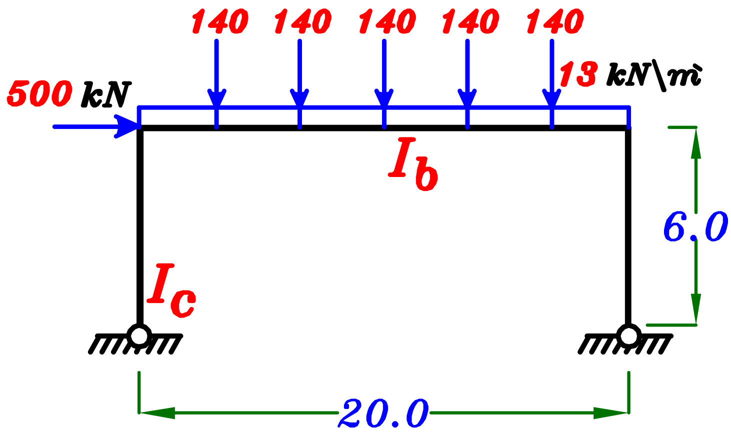
$$\boxed{\delta_{1o} + X_1 \ \delta_{11} = Zero}$$ Get X_1

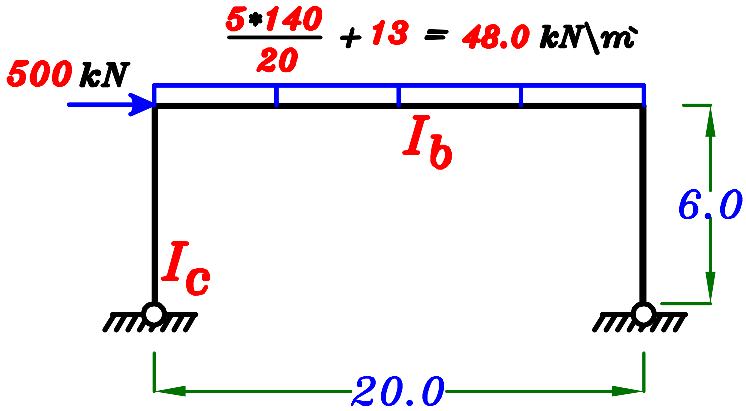


$$\boxed{M_F = M_o + X_1 M_1}$$

Example.

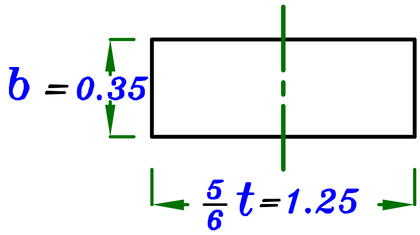


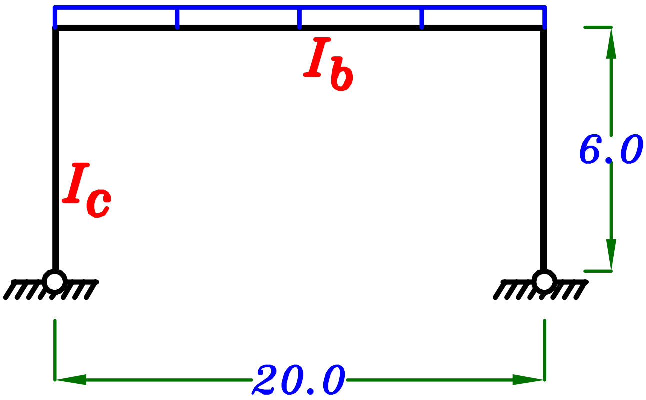




Using Virtual Work.

I_c



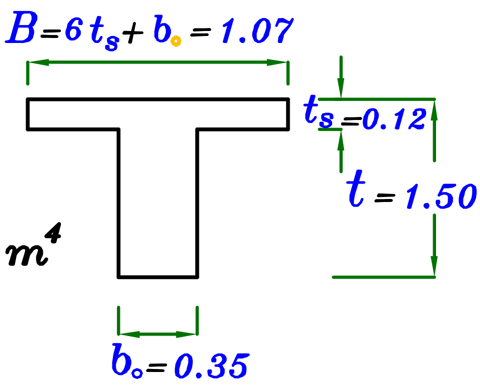


$$I_c = \frac{b\left(\frac{5}{6}t\right)^3}{12} = \frac{0.35\left(\frac{5}{6}*1.50\right)^3}{12}$$

$$= 0.0569 \ m^4$$

I_b

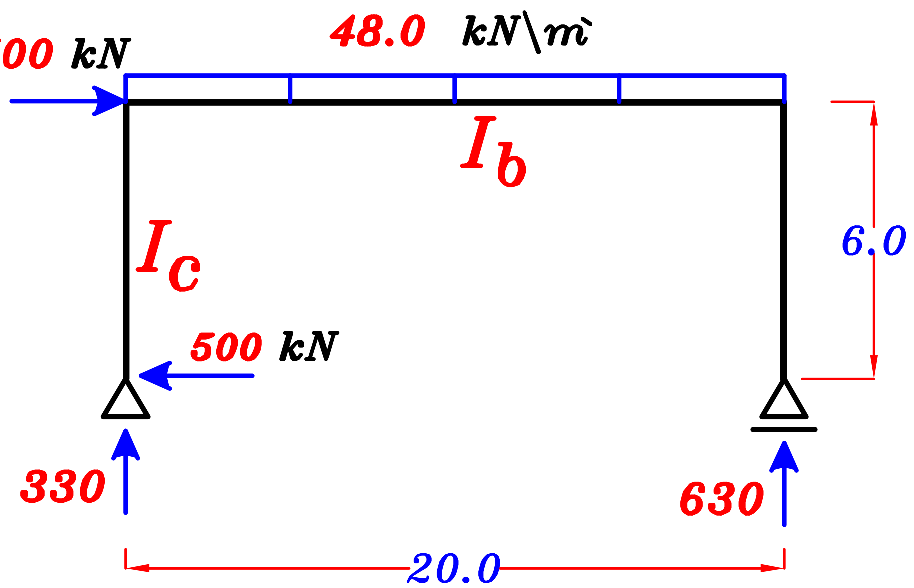
$$\left.\begin{aligned} \frac{t_s}{t} &= \frac{0.12}{1.50} = 0.08 \\ \frac{b_o}{B} &= \frac{0.35}{1.07} = 0.327 \end{aligned}\right\} \quad \text{Table page 63} \quad \mu = 362.4$$

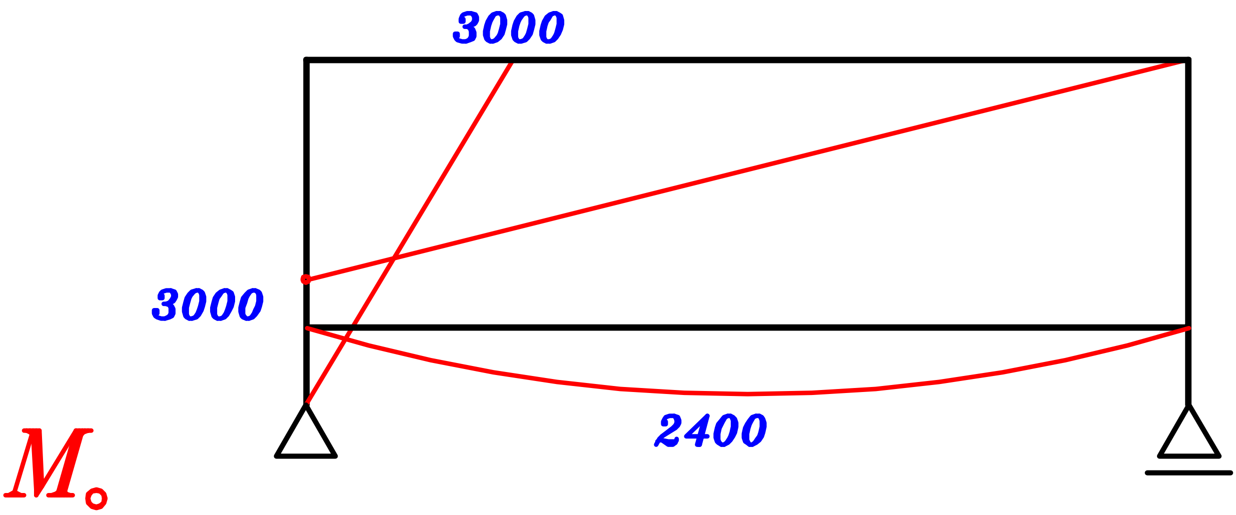


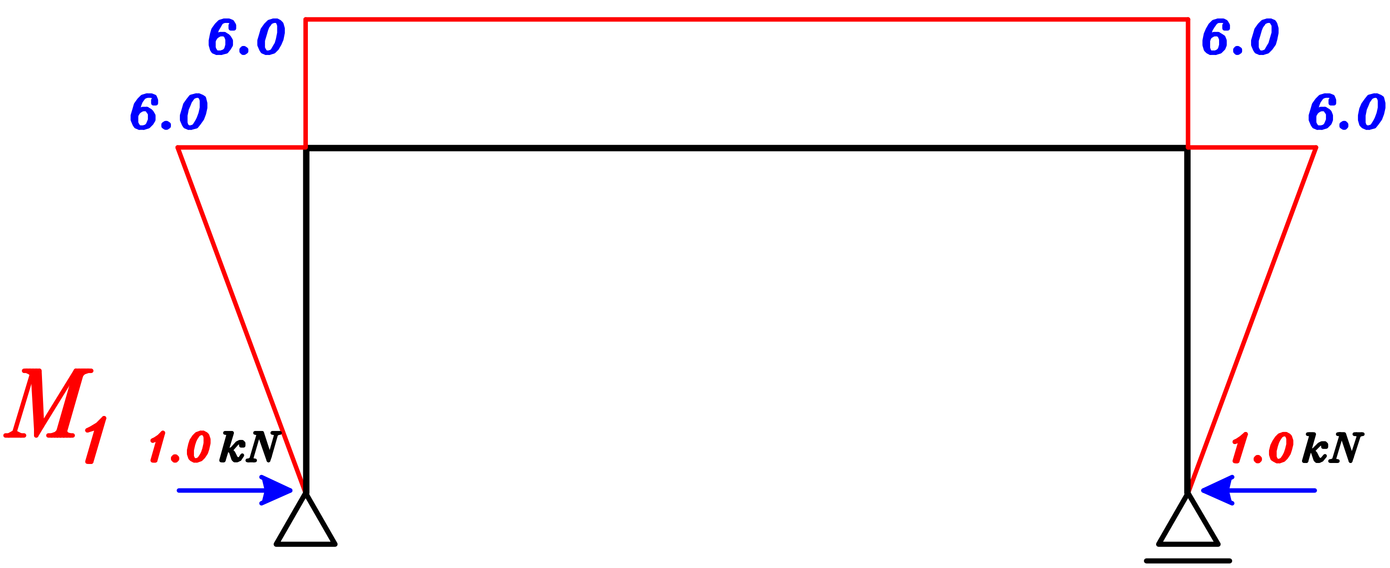
$$I_b = (\mu * 10^{-4})\ B\,t^3 = 362.4 * 10^{-4} * 1.07 * 1.50^3 = 0.1308 \ m^4$$

$$\therefore \boxed{I_b = 2.297\, I_c}$$

Using Virtual Work Method.







$$\delta_{1o} = \frac{1}{E_c\, I_b} * (M_o * M_1) + \frac{1}{E_c\, I_c} * (M_o * M_1)$$

$$\delta_{1o} = \frac{-1}{E_c\,(2.297)\,I_c}\left(\frac{1}{2}(20.0)(3000)[6.0] + \frac{2}{3}(2400)(20)[6.0]\right)$$

$$\frac{-1}{E_c\, I_c}\left(\frac{1}{2}(6)(3000)\left[\frac{2}{3} * 6.0\right]\right) = \frac{-197950.37}{E_c\, I_c}$$

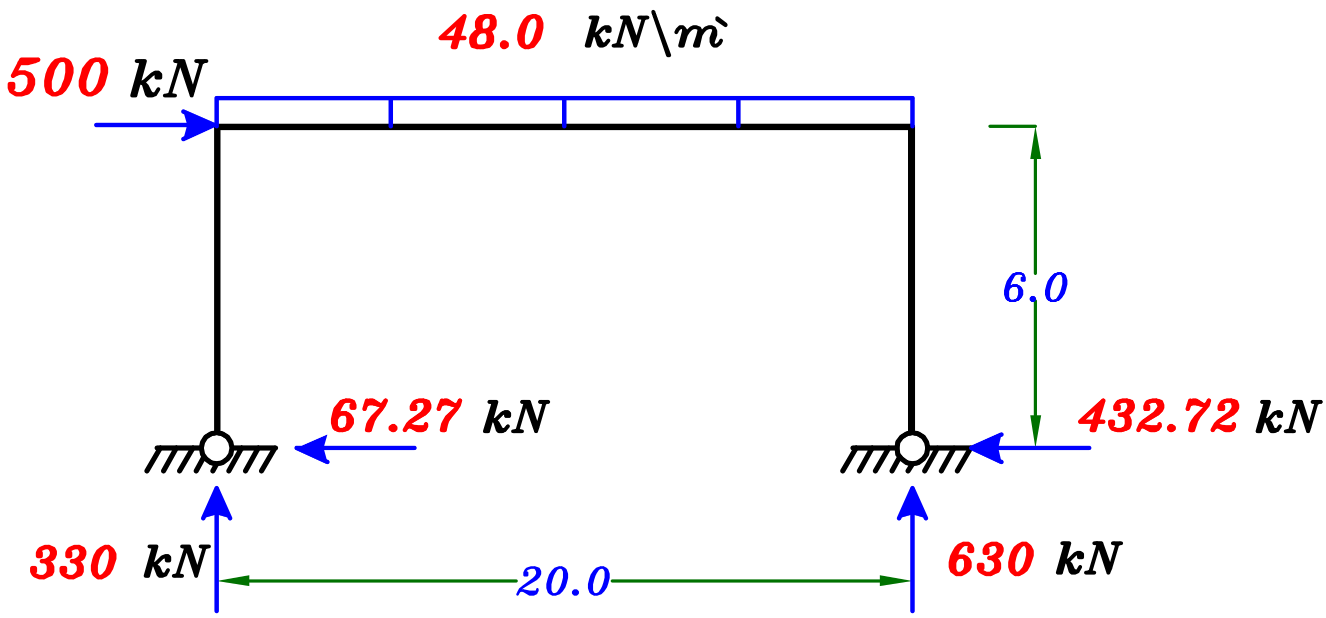
$$\delta_{11} = \frac{1}{E_c\, I_b} * (M_1 * M_1) + \frac{2}{E_c\, I_c} * (M_1 * M_1)$$

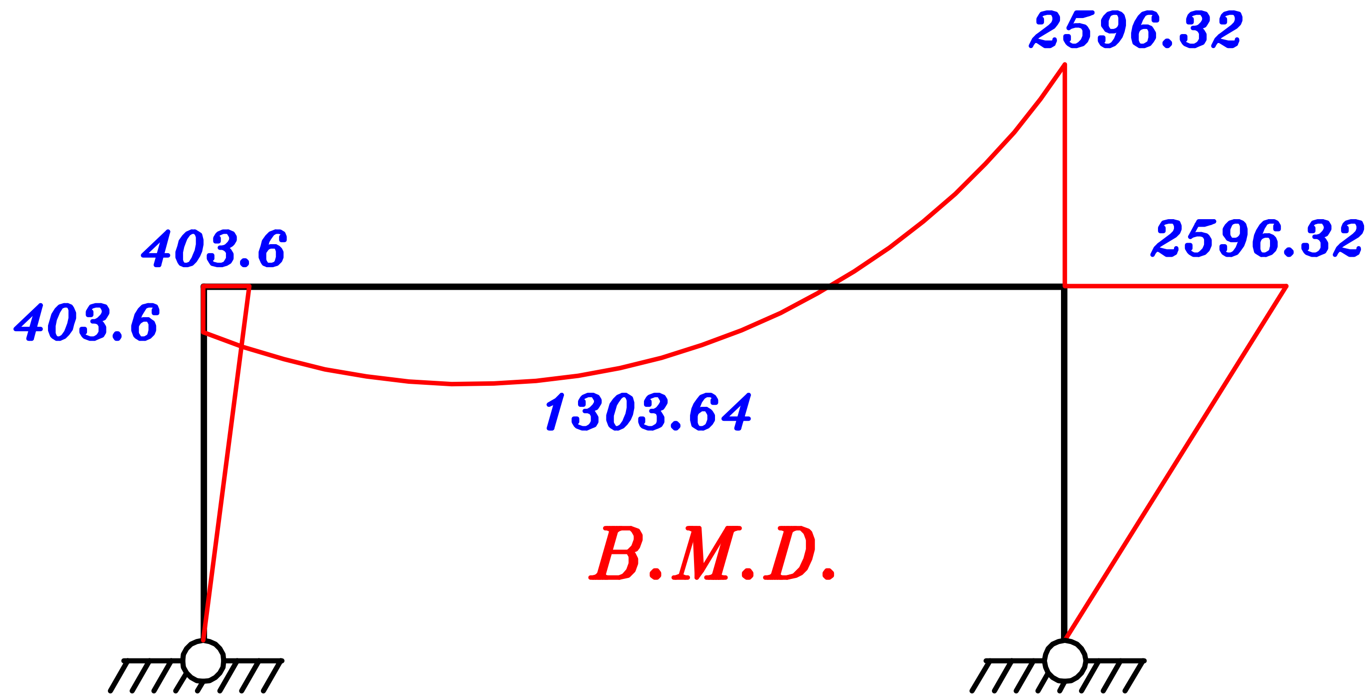
$$\delta_{11} = \frac{1}{E_c\,(2.297)\,I_c}\left((6.0)(20.0)[6.0]\right) + \frac{2}{E_c\, I_c}\left(\frac{1}{2}(6.0)(6.0)\left[\frac{2}{3} * 6.0\right]\right)$$

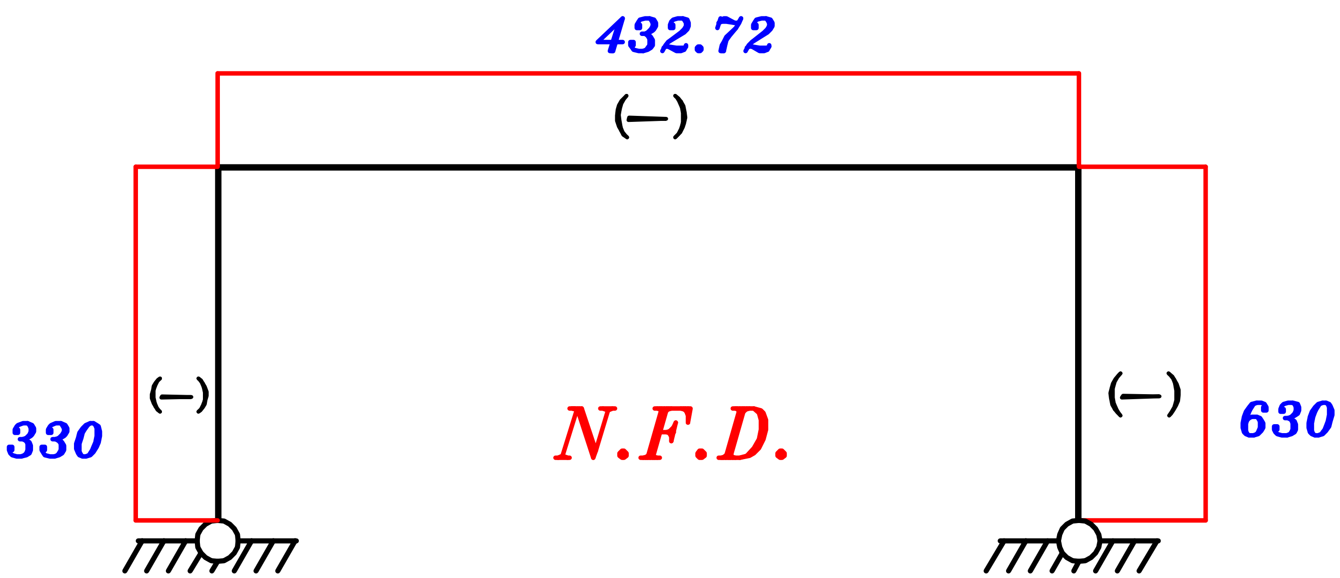
$$= \frac{457.45}{E_c\, I_c}$$

$$\because \quad \delta_{1o} + X\ \delta_{11} = Zero$$

$$\frac{-197950.37}{E_c\, I_c} + X * \frac{457.45}{E_c\, I_c} = Zero \longrightarrow \boxed{X = 432.72\ kN}$$



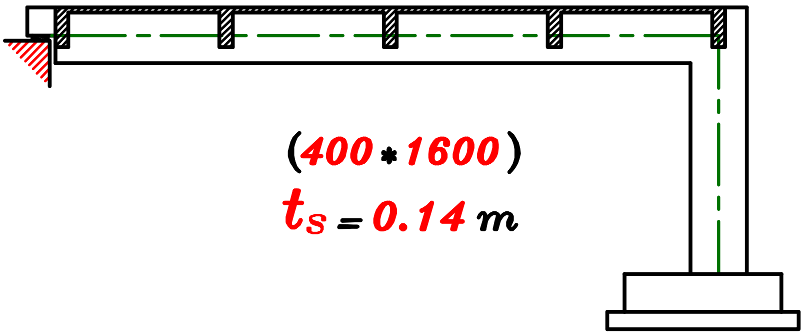

2596.32
403.6
2596.32
403.6
1303.64
B.M.D.

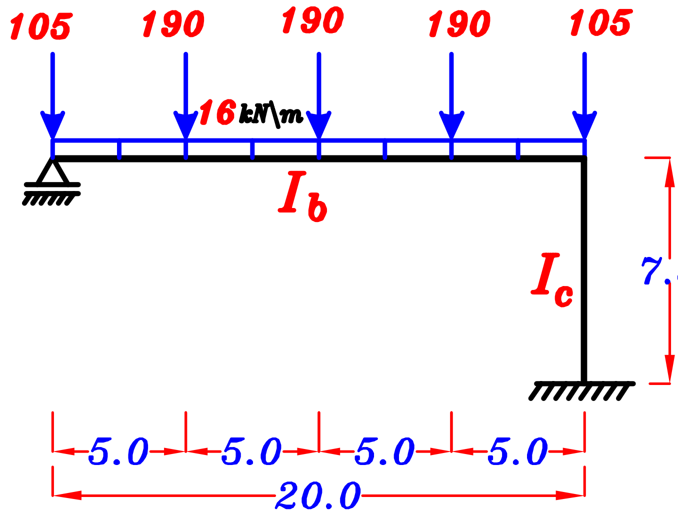

432.72
(—)
(—)
(—)
330
N.F.D.
630

Roller–Fixed Frame.

Example.

For the given Frame, Draw B.M.D. & N.F.D.





For the Fixed Roller Frame.

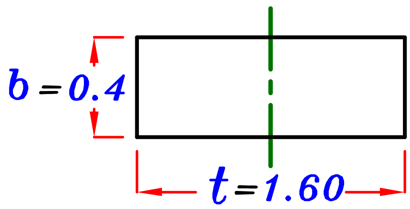
we will use Virtual Work Method.

Solution.

I_c

$$I_c = \frac{b\,(t)^3}{12} = \frac{0.4\,(1.60)^3}{12}$$

$$= 0.1365 \ m^4$$

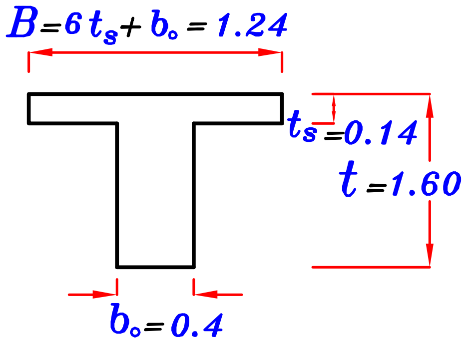


I_b

$$\frac{t_s}{t} = \frac{0.14}{1.60} = 0.0875$$

$$\frac{b_o}{B} = \frac{0.4}{1.24} = 0.322$$

Table Page 63 $\mu = 368$



$$I_b = (\mu * 10^{-4})\, B\, t^3 = 368 * 10^{-4} * 1.24 * 1.60^3$$

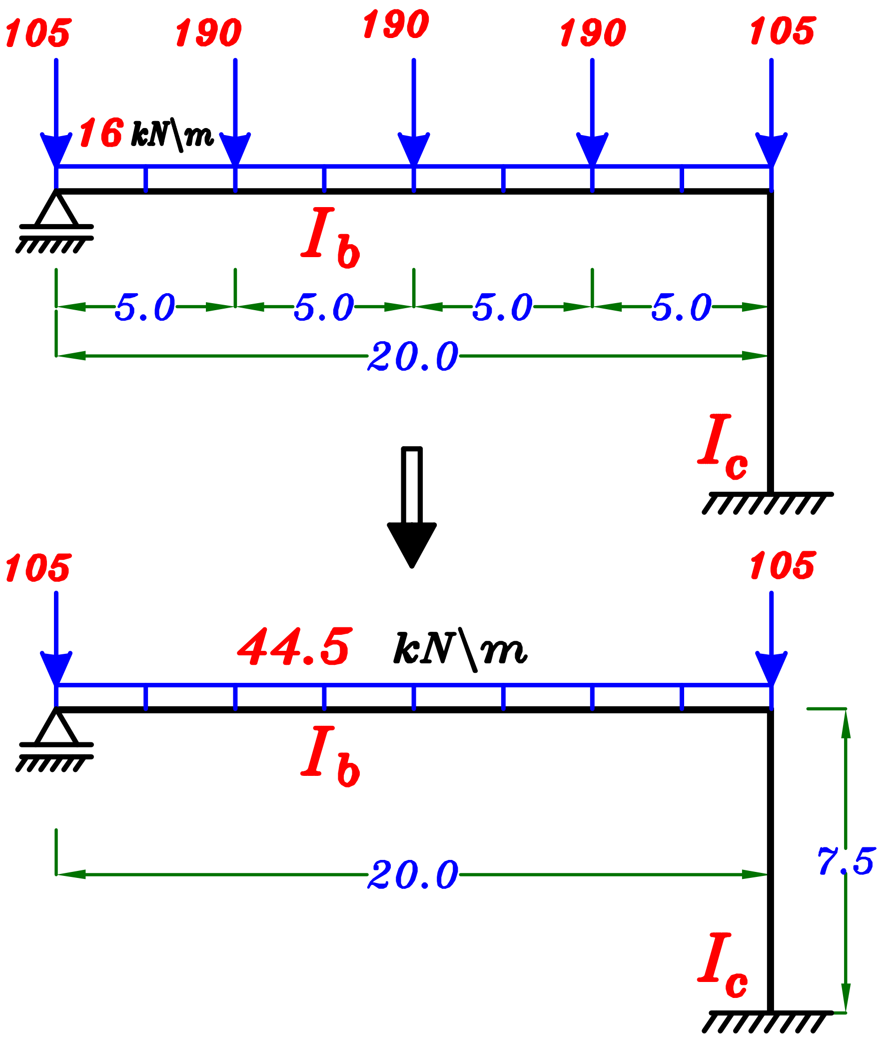
$$= 0.18690 \ m^4$$

$$\therefore \ \boxed{I_b = 1.37\ I_c}$$

Concentrated loads للتسهيل يتم تحويل الاحمال المركزه

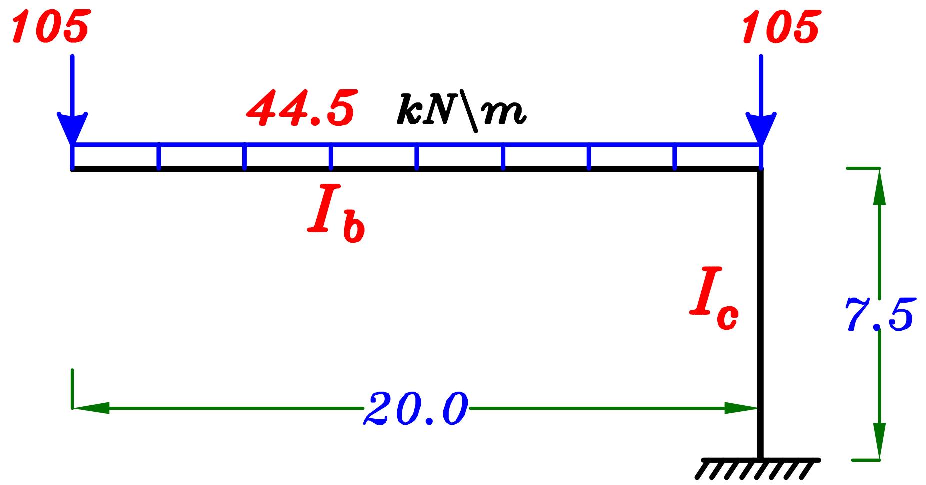
Distibuted loads الى احمال منتظمه

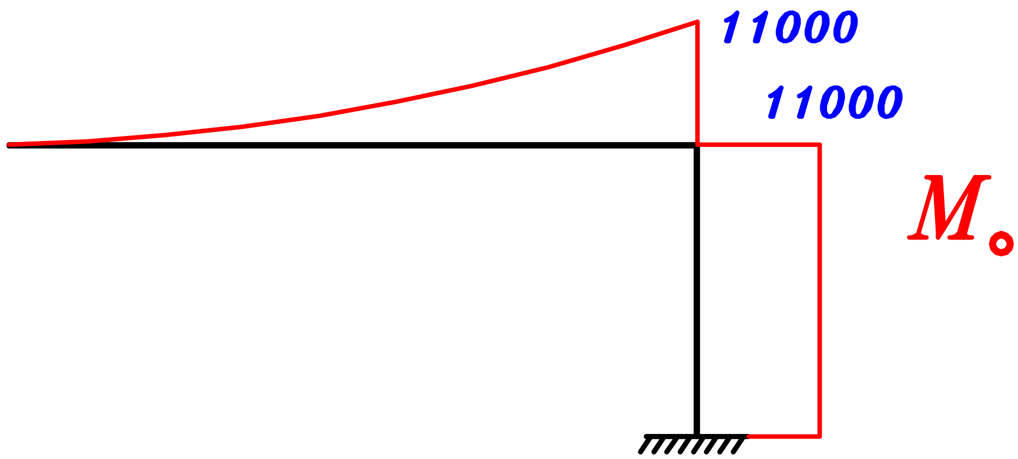
$$w = o.w. + \frac{\sum P}{span} = 16.0 + \frac{3\,(190)}{20.0} = 44.5 \ kN\backslash m`$$

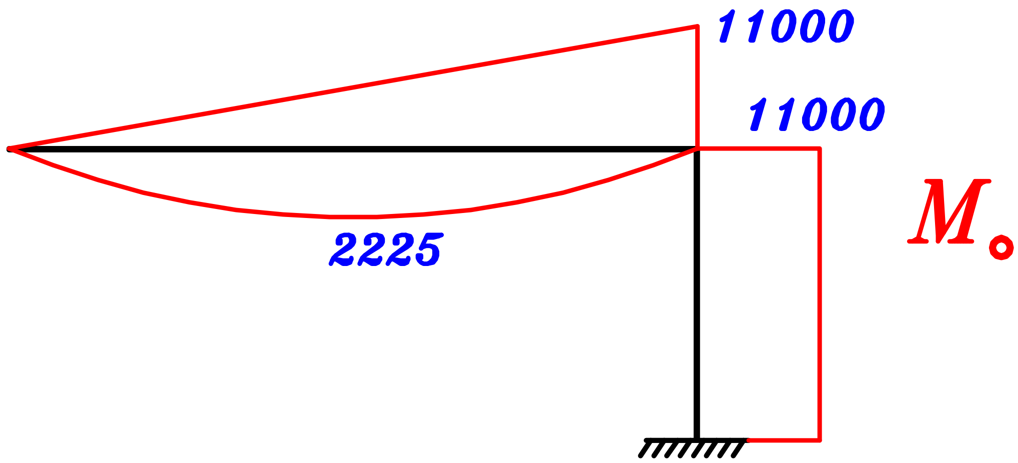


Determinate and stable نحول الشكل الى M_o لرسم

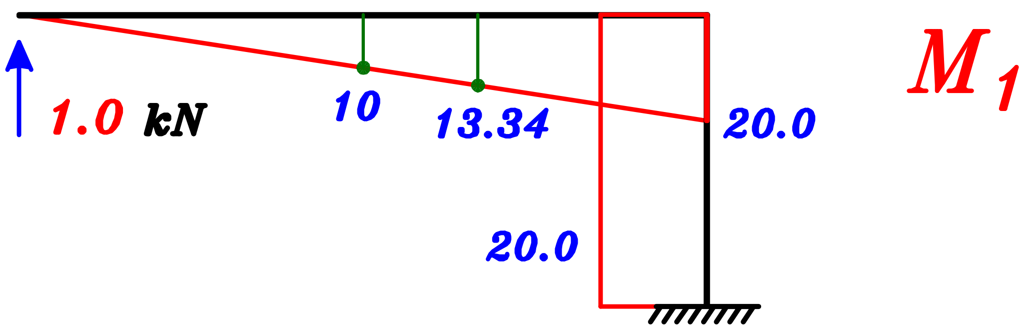
ⓑ Make the Frame Determinate and stable then draw B.M.D. ((M_o))







نحذف كل الاحمال و نضع 1.0 kN فى اتجاه المجهول



$$\delta_{10} = \frac{1}{E_c I_c} * (M_o * M_1) + \frac{1}{E_c I_b} * (M_o * M_1)$$

$$\delta_{10} = \frac{-1}{E_c I_c}\Big((11000)(7.5)[20]\Big) + \frac{-1}{E_c (1.37) I_c}\Big(\tfrac{1}{2}(11000)(20)[13.34]\Big)$$

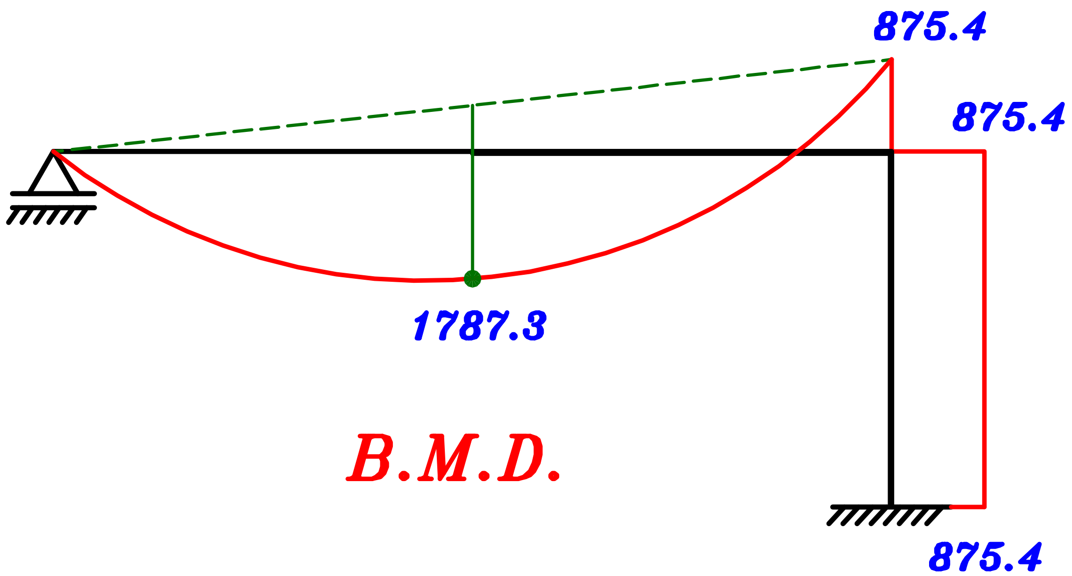
$$+ \frac{1}{E_c (1.37) I_c}\Big(\tfrac{2}{3}(2225)(20)\ [10]\Big) = \frac{-2504550}{E_c I_c}$$

$$\delta_{11} = \frac{1}{E_c I_c} * (M_1 * M_1) + \frac{1}{E_c I_b} * (M_1 * M_1)$$

$$\delta_{11} = \frac{1}{E_c I_c}\Big((20)(7.5)[20]\Big) + \frac{1}{E_c (1.37) I_c}\Big(\tfrac{1}{2}(20)(20)[13.34]\Big) = \frac{4947.44}{E_c I_c}$$

$$\because\ \delta_{10} + Y\ \delta_{11} = Zero$$

$$= \frac{-2504550}{E_c I_c} + Y * \frac{4947.44}{E_c I_c} \longrightarrow \boxed{Y = 506.23\ kN}$$

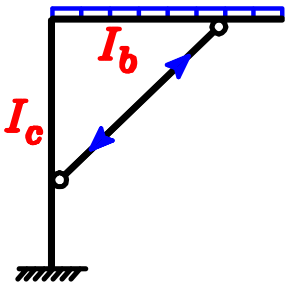


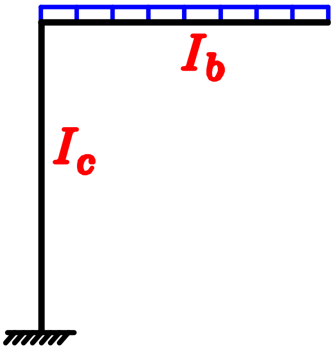
B.M.D.

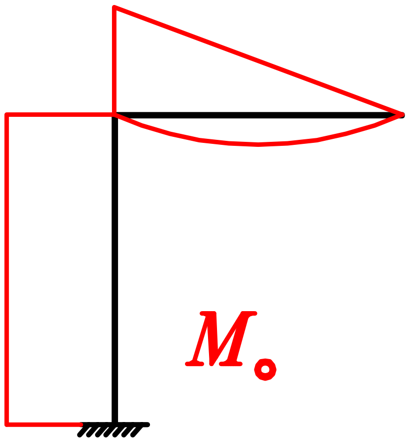
N.F.D.

Cantilever Frame with link member.

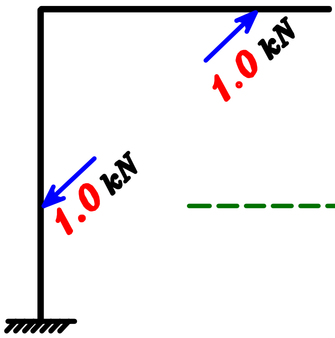
@IF the Link member is Compression member.

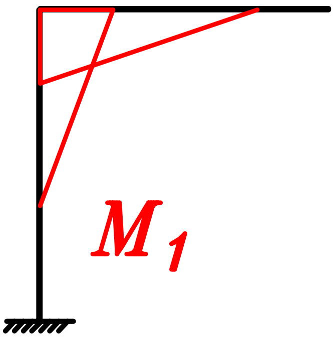






M_o



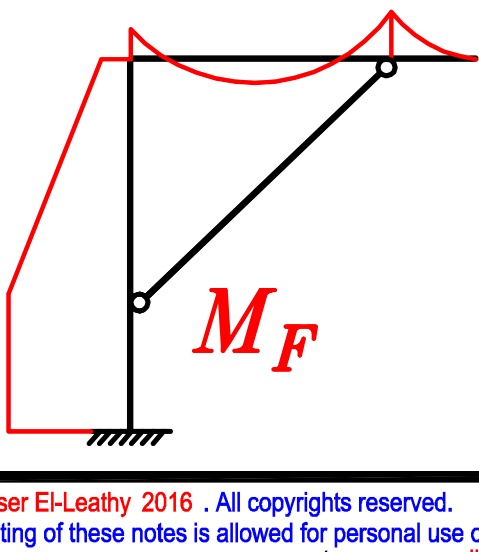


M_1

$$\delta_{1o} = \frac{1}{E_c I_b} * (M_o * M_1) + \frac{1}{E_c I_c} * (M_o * M_1)$$

$$\delta_{11} = \frac{1}{E_c I_b} * (M_1 * M_1) + \frac{1}{E_c I_c} * (M_1 * M_1)$$

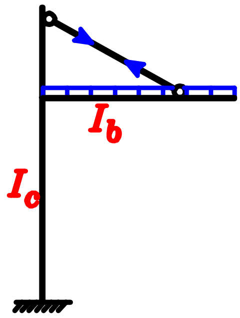
$\boxed{\delta_{1o} + X\,\delta_{11} = Zero}$ Get X $\boxed{M_F = M_o + X\, M_1}$



M_F

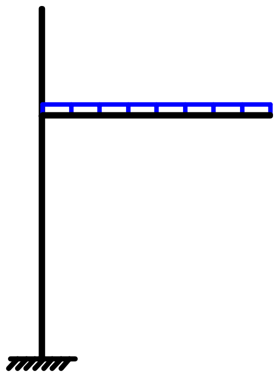
ⓑ IF the Link member is Tension member.

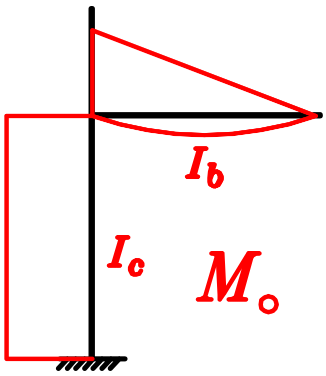
we will take the extension of Tie into consideration.

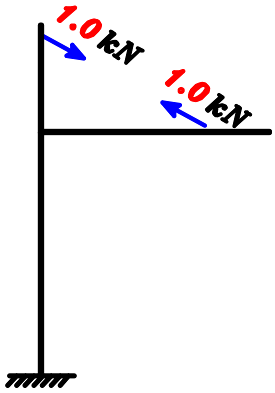


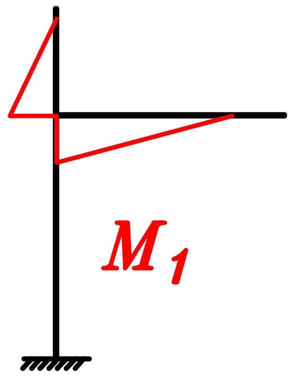
هذه الخطوه ممكن إهمالها للتسهيل إلا مع

Polygon Frames & Arch Girder









$$\delta_{1o} = \frac{1}{E_c I_b} * (M_o * M_1) + \frac{1}{E_c I_c} * (M_o * M_1) \qquad \boxed{E_c = 4400\sqrt{F_{cu}}} \; N/mm^2$$

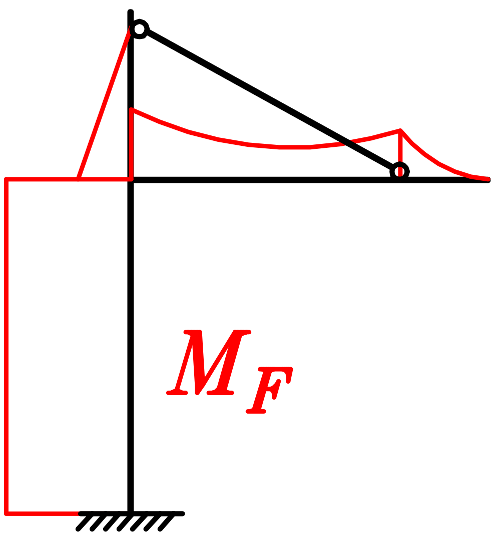
$$\delta_{11} = \frac{1}{E_c I_b} * (M_1 * M_1) + \frac{1}{E_c I_c} * (M_1 * M_1) \qquad \boxed{E_s = 2.10 * 10^{-5}} \; N/mm^2$$

$$\Delta_{Tie} = \frac{F_s * L}{E_s} \; (working) \quad , \quad \Delta_{Tie} = \frac{(F_y \backslash \delta_s) * L}{E_s} \; (U.L.)$$

$L =$ Length of the Tie.

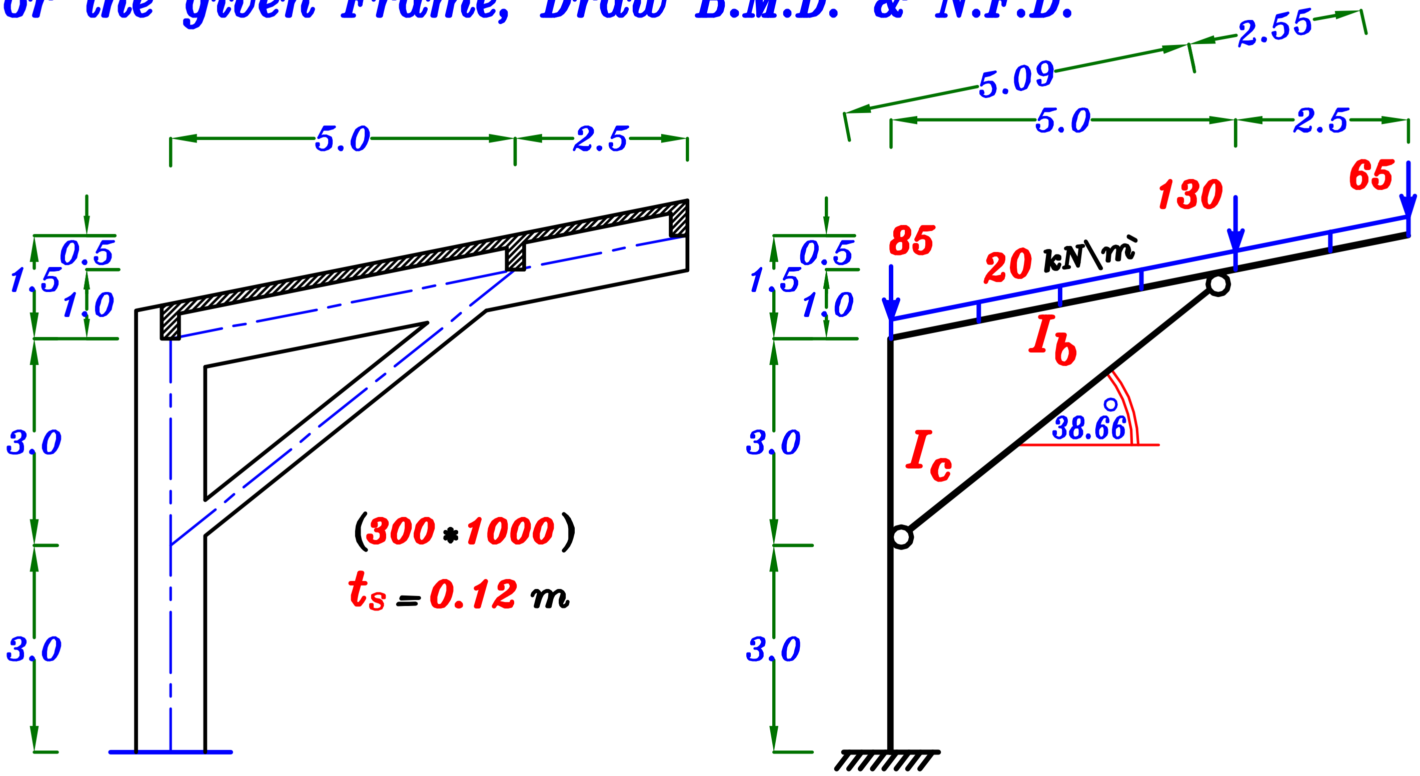
$$\boxed{F_y = 360 \; N\backslash mm^2 = 360 * 10^3 \; kN\backslash m^2}$$

$$\boxed{\delta_{1o} + X\,\delta_{11} + \Delta_{Tie} = Zero} \quad \text{Get } X \quad \boxed{M_F = M_o + X\,M_1}$$



Example.

For the given Frame, Draw B.M.D. & N.F.D.



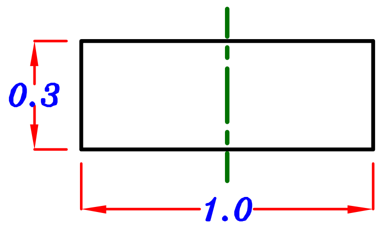
For the Cantilever Frame with Link member.
we will use Virtual Work Method.

Solution.

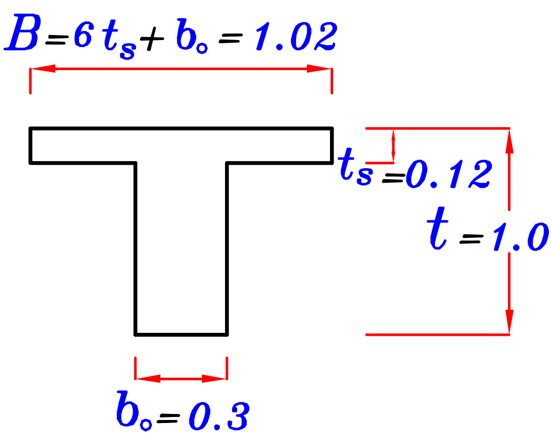
$\underline{\underline{I_c}}$

$$I_c = \frac{b(t)^3}{12} = \frac{0.3(1.0)^3}{12}$$

$$= 0.025 \ m^4$$



$\underline{\underline{I_b}}$

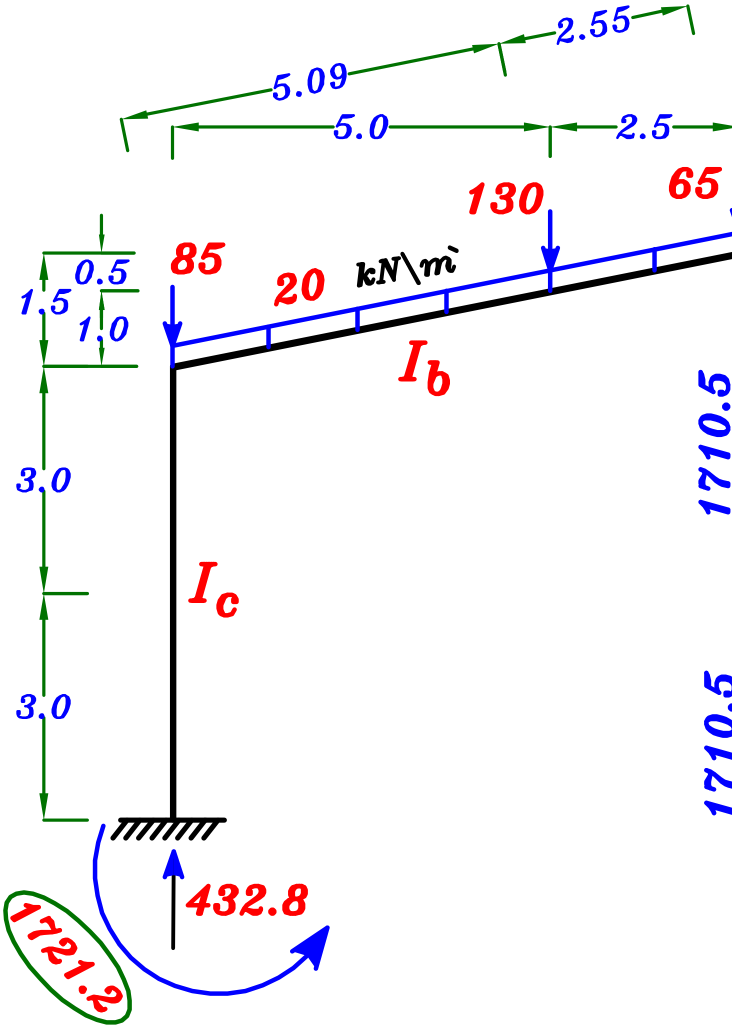


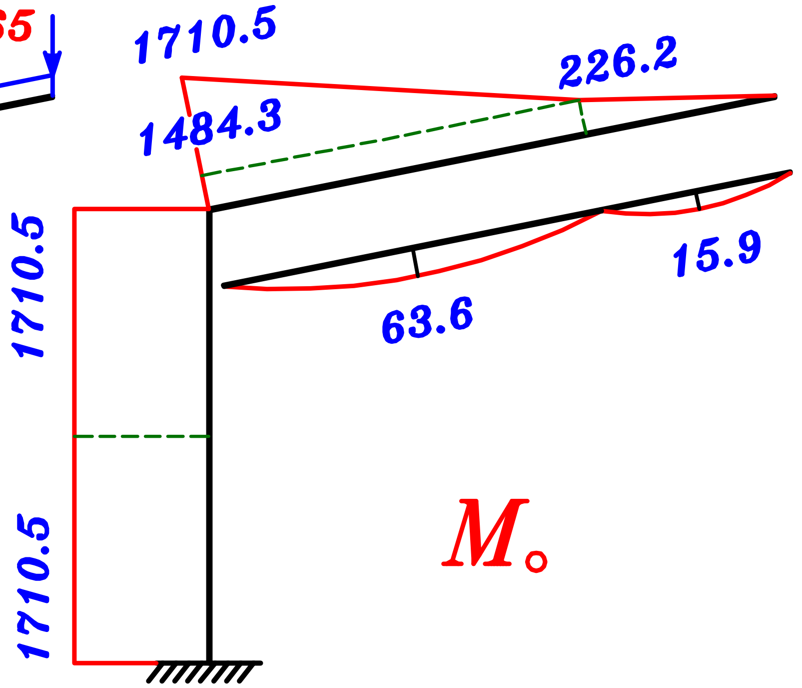
$$\left.\begin{aligned} \frac{t_s}{t} &= \frac{0.12}{1.0} = 0.12 \\ \frac{b_o}{B} &= \frac{0.3}{1.02} = 0.294 \end{aligned}\right\} \quad \text{Table Page 63} \quad \mu = 361$$

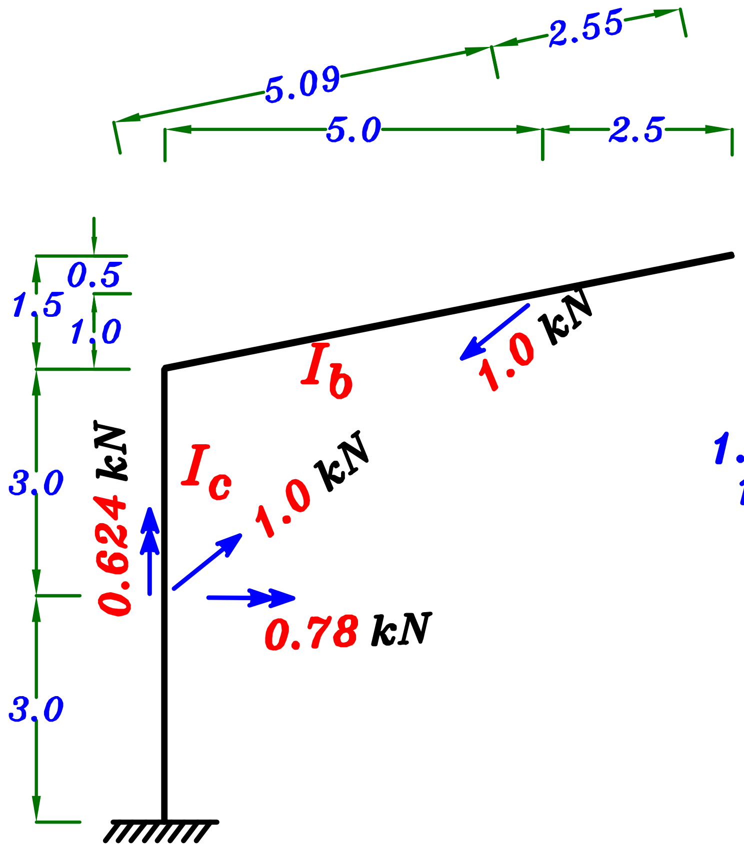
$$I_b = (\mu * 10^{-4}) B t^3 = 361 * 10^{-4} * 1.02 * 1.0^3$$

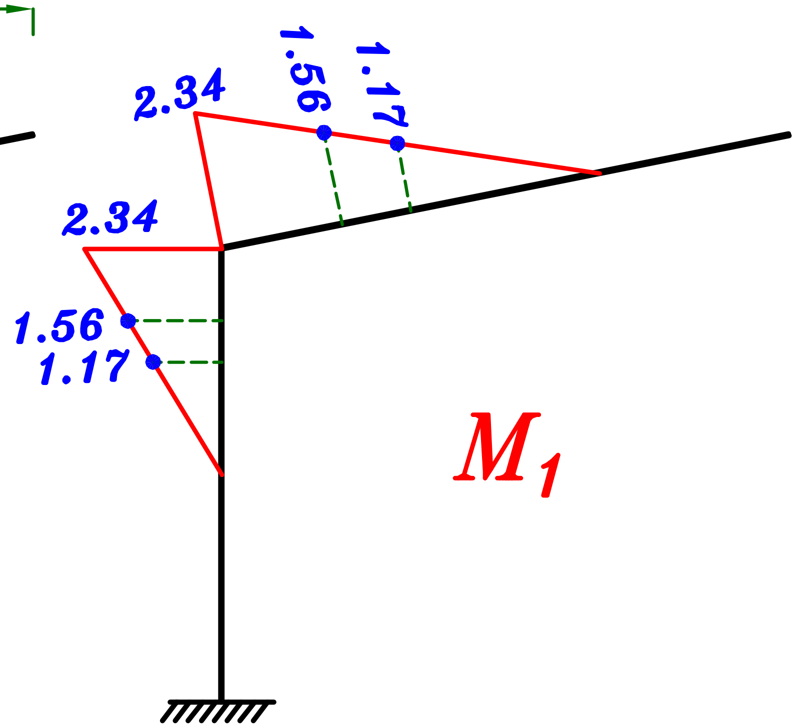
$$= 0.0368 \ m^4$$

$$\therefore \boxed{I_b = 1.472 \, I_c}$$


5.09
2.55
5.0
2.5
130
65
85
20
kN\m`
0.5
1.5
1.0
Ib
3.0
Ic
3.0
432.8
1721.2


1710.5
226.2
1484.3
1710.5
15.9
63.6
1710.5
M∘


5.09
2.55
5.0
2.5
0.5
1.5
1.0
Ib
1.0 kN
3.0
0.624 kN
Ic
1.0 kN
0.78 kN
3.0


2.34
1.56
1.17
2.34
1.56
1.17
M1

$$\delta_{1\circ} = \frac{1}{E_c I_c} * (M_\circ * M_1) + \frac{1}{E_c I_b} * (M_\circ * M_1)$$

$$\delta_{1\circ} = \frac{1}{E_c I_c} \Big((1710.5)(3.0)[1.17] \Big) + \frac{1}{E_c (1.472) I_c} \Big(\tfrac{1}{2}(5.09)(1484.3)[1.56]$$

$$+ (5.09)(226.2)[1.17] - \tfrac{2}{3}(63.6)(5.09)[1.17] \Big) = \frac{+10750.8}{E_c I_c}$$

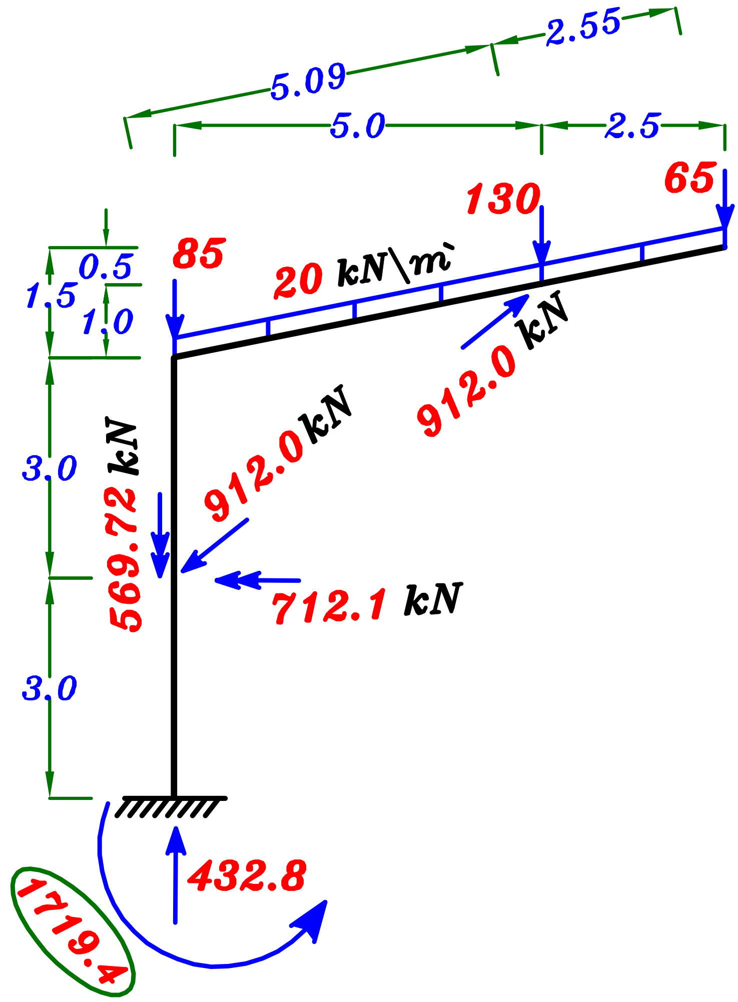
$$\delta_{11} = \frac{1}{E_c I_c} * (M_1 * M_1) + \frac{1}{E_c I_b} * (M_1 * M_1)$$

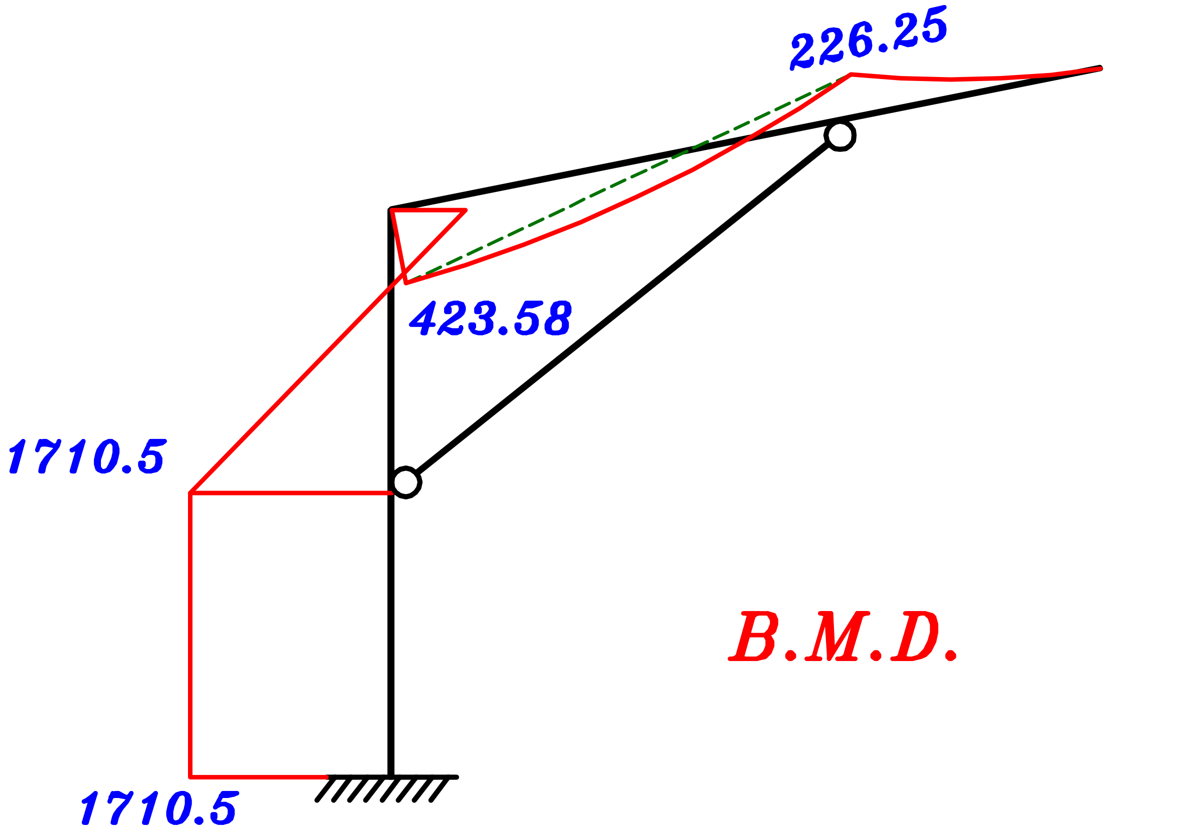
$$\delta_{11} = \frac{1}{E_c I_c} \Big(\tfrac{1}{2}(2.34)(3.0)[1.56] \Big) + \frac{1}{E_c (1.472) I_c} \Big(\tfrac{1}{2}(2.34)(5.09)[1.56] \Big)$$

$$= \frac{11.787}{E_c I_c}$$

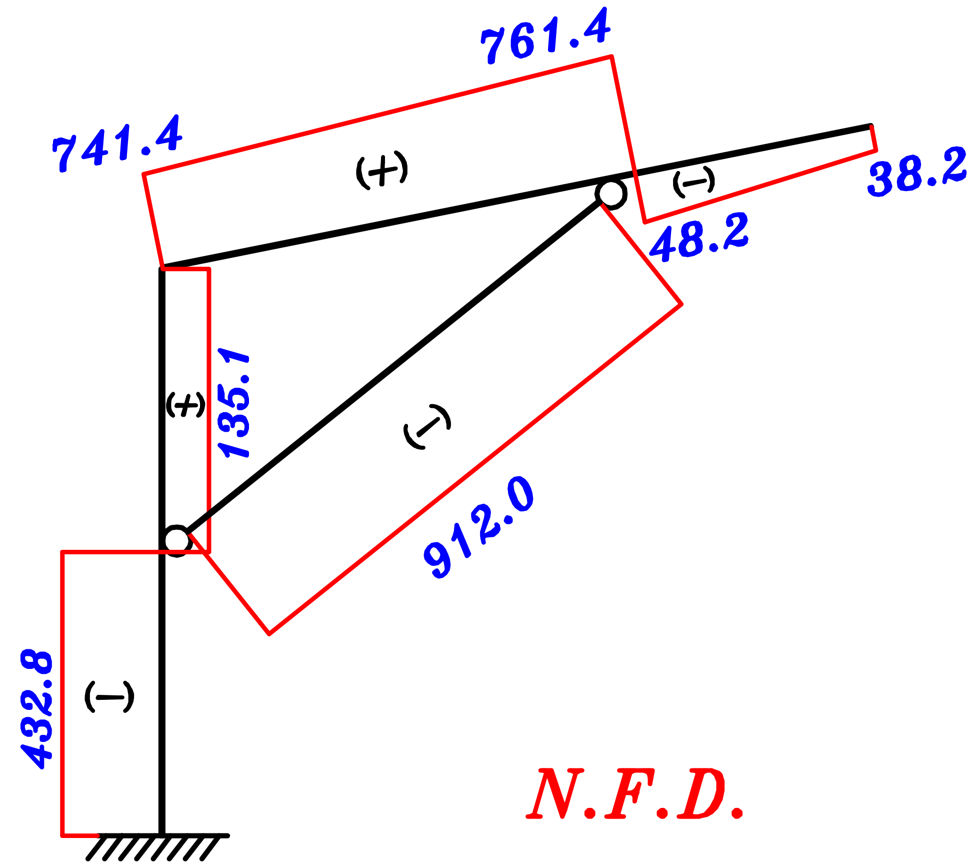
$$\because \quad \delta_{1\circ} + X \ \delta_{11} = Zero$$

$$= \frac{+10750.8}{E_c I_c} + X * \frac{11.787}{E_c I_c} \longrightarrow \boxed{X = -\ 912.0 \ kN}$$





226.25
423.58
1710.5
1710.5
B.M.D.



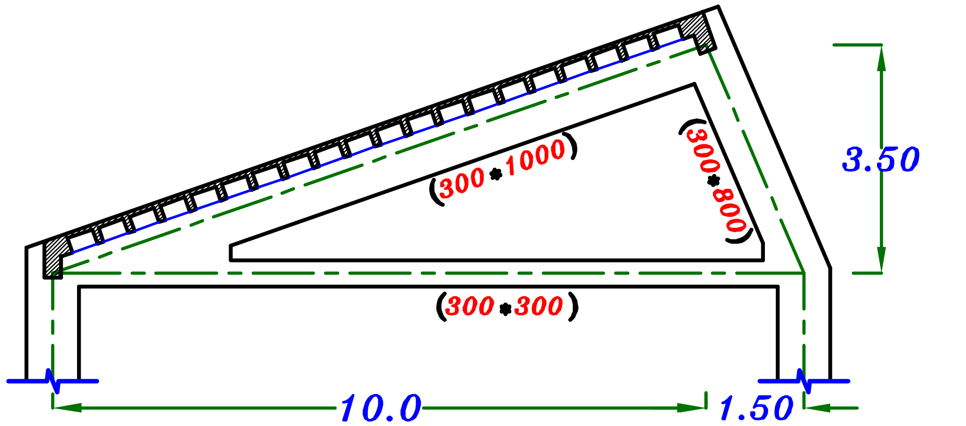
761.4
741.4
(+)
(–)
38.2
48.2
(+)
135.1
(–)
912.0
432.8
(–)
N.F.D.

Saw Tooth Girder Type.

Example.

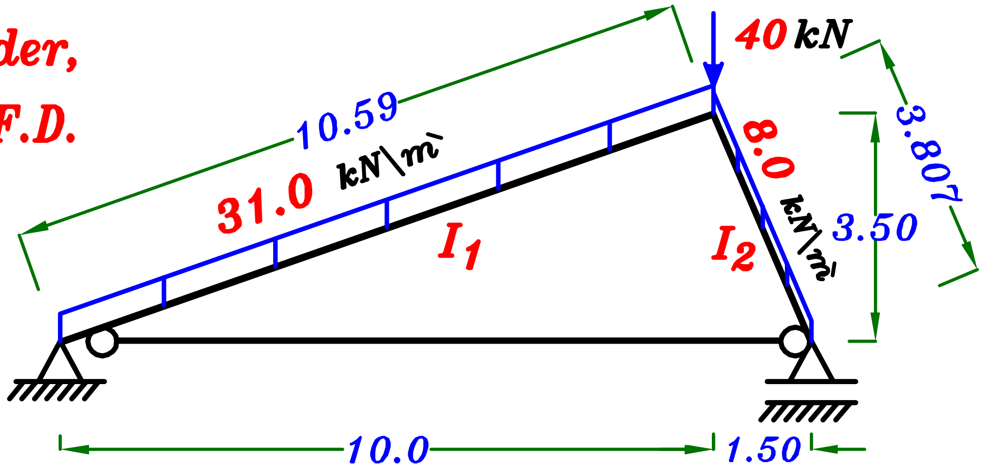
$t_{(H.B.)} = 30$ cm.

(U.L.) Loads



For the given Girder,

Draw B.M.D. & N.F.D.



For the Saw Tooth (Girder type).

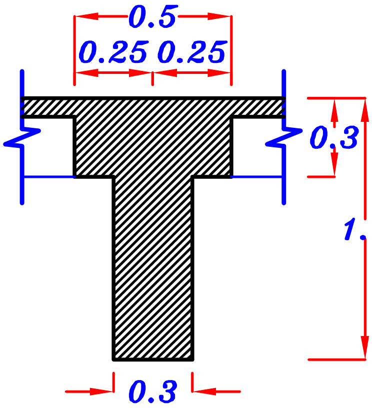
we will use Virtual Work Method.

Solution.

$$I_1 = (\mu * 10^{-4})\, B\, t^3$$

$b = 0.30$ m , $t_s = 0.30$ m

$B = 0.60$ m , $t = 1.0$ m

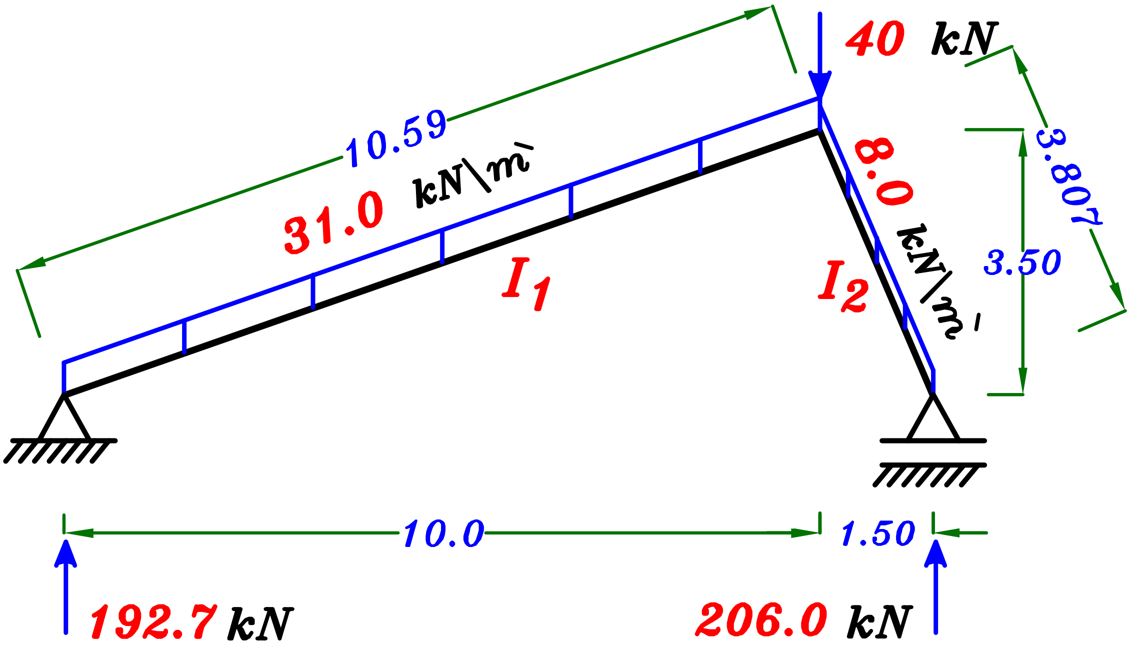


$$\left.\begin{array}{l} \dfrac{t_s}{t} = \dfrac{0.30}{1.0} = 0.30 \\[2ex] \dfrac{b}{B} = \dfrac{0.30}{0.50} = 0.60 \end{array}\right\} \begin{array}{l} \text{Table Page 63} \\ \mu = 631 \end{array}$$

$$I_1 = (\mu * 10^{-4})\, B\, t^3 = (631 * 10^{-4} * 0.50 * 1.0^3) = 0.03155 \text{ m}^4$$

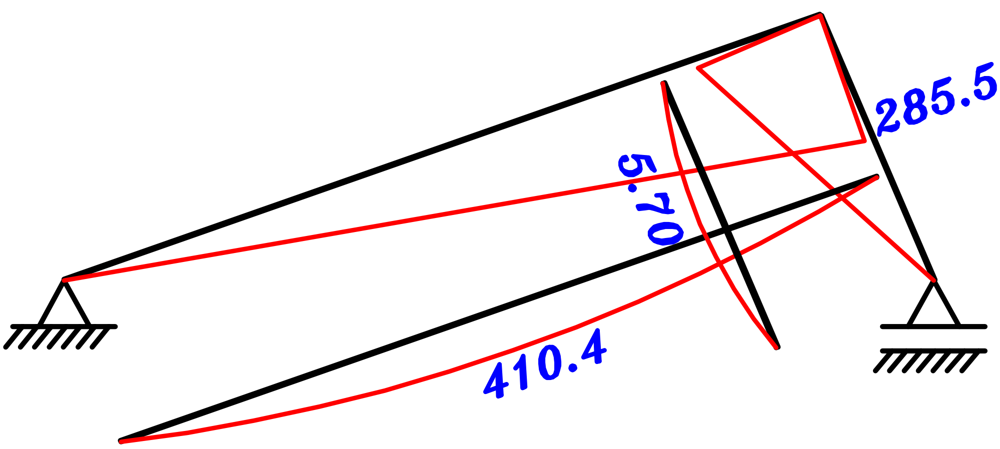
$$I_2 = \frac{b\,(t)^3}{12} = \frac{0.30\,(0.80)^3}{12} = 0.0128 \text{ m}^4$$

$$\therefore \boxed{I_1 = 2.465\, I_2}$$



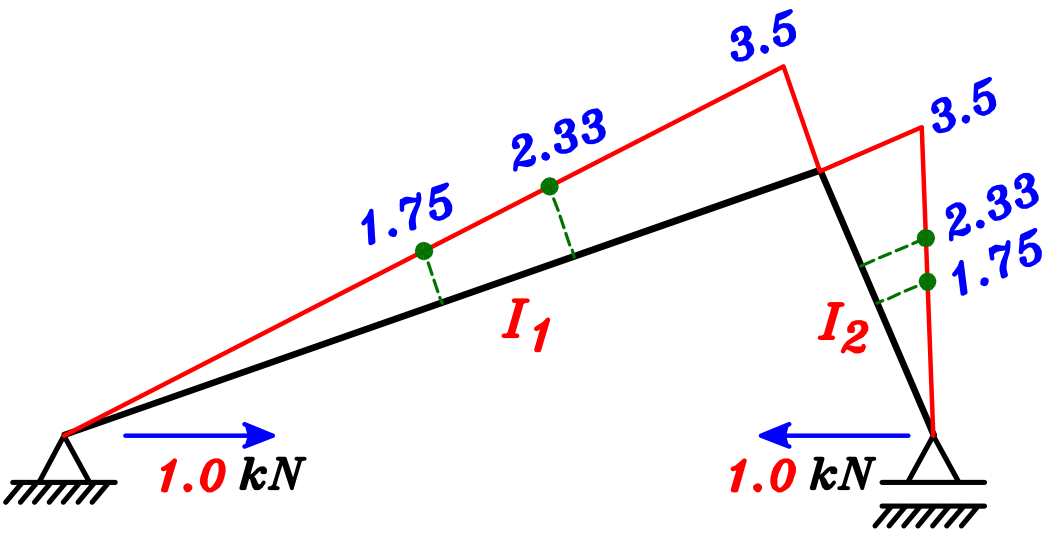
40 kN
10.59
31.0 kN\m`
8.0 kN\m`
3.807
3.50
I1
I2
10.0
1.50
192.7 kN
206.0 kN

$M_\circ$



285.5
5.70
410.4

M_1



3.5
3.5
2.33
1.75
2.33
1.75
I1
I2
1.0 kN
1.0 kN

$$\delta_{10} = \frac{1}{E_c I_1} * (M_\circ * M_1) + \frac{1}{E_c I_2} * (M_\circ * M_1)$$

$$\delta_{10} = \frac{-1}{E_c (2.465) I_2} \left(\tfrac{1}{2}(10.59)(285.5)[2.33] + \tfrac{2}{3}(410.4)(10.59)[1.75] \right)$$

$$+ \frac{-1}{E_c I_2} \left(\tfrac{1}{2}(3.807)(285.5)[2.33] + \tfrac{2}{3}(5.70)(3.807)[1.75] \right) = \frac{-4777.48}{E_c I_2}$$

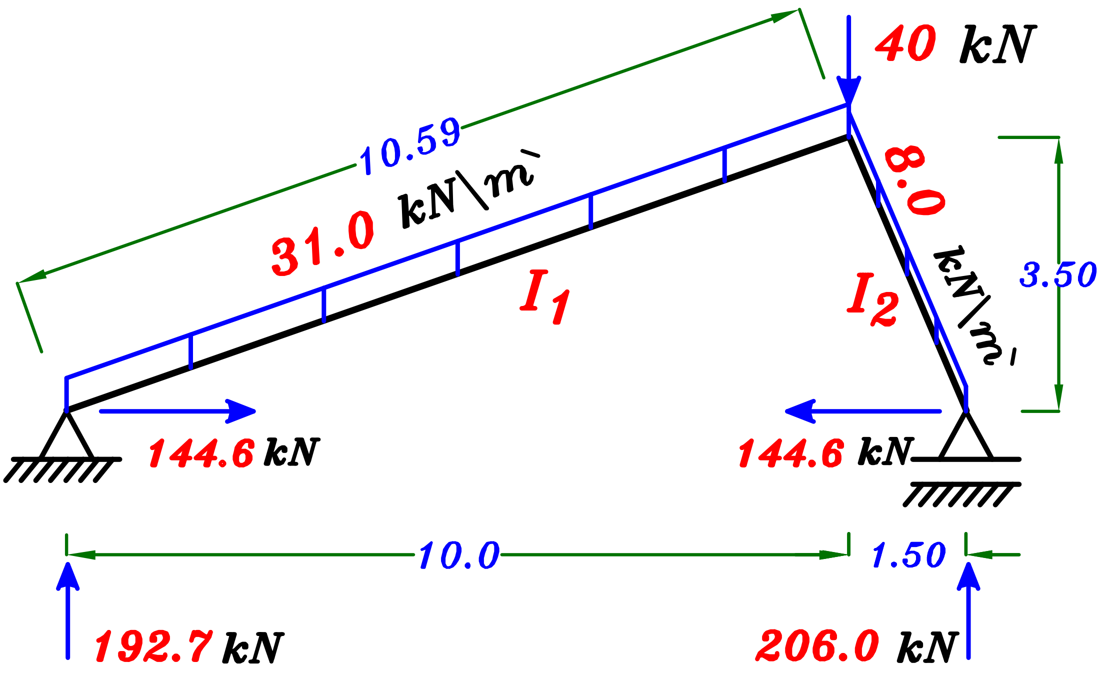
$$\delta_{11} = \frac{1}{E_c I_1} * (M_1 * M_1) + \frac{1}{E_c I_2} * (M_1 * M_1)$$

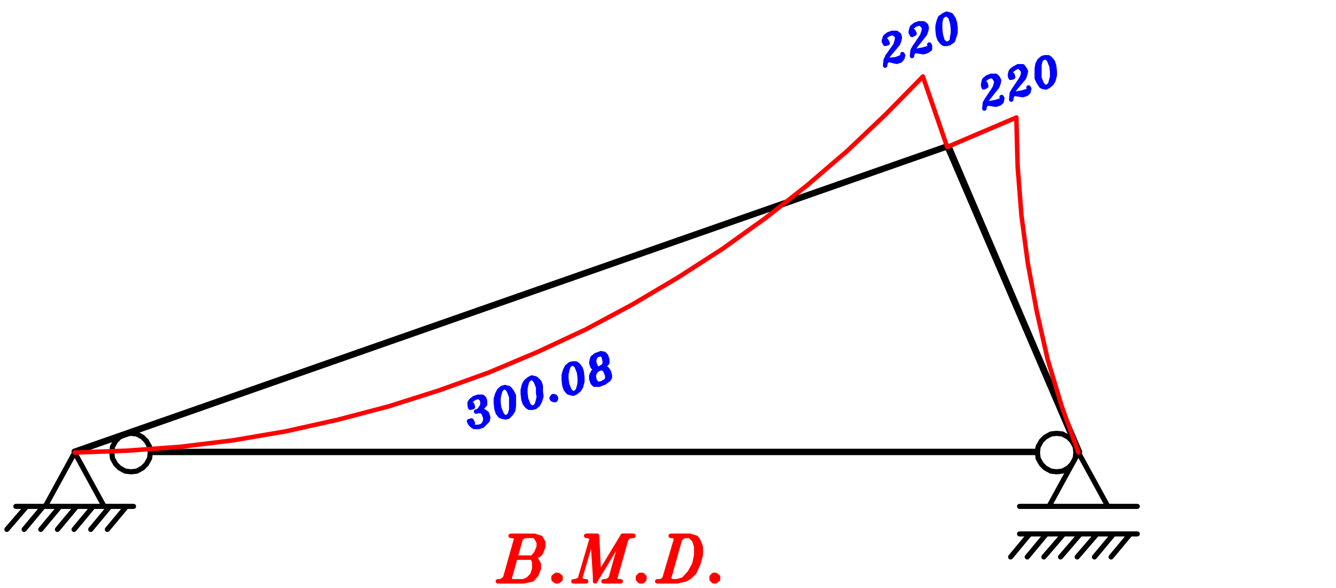
$$\delta_{11} = \frac{1}{E_c (2.465) I_2} \left(\tfrac{1}{2}(10.59)(3.5)[2.33] \right) + \frac{1}{E_c I_2} \left(\tfrac{1}{2}(3.807)(3.5)[2.33] \right)$$

$$= \frac{17.51}{E_c I_2} + \frac{15.523}{E_c I_2} = \frac{33.033}{E_c I_2}$$

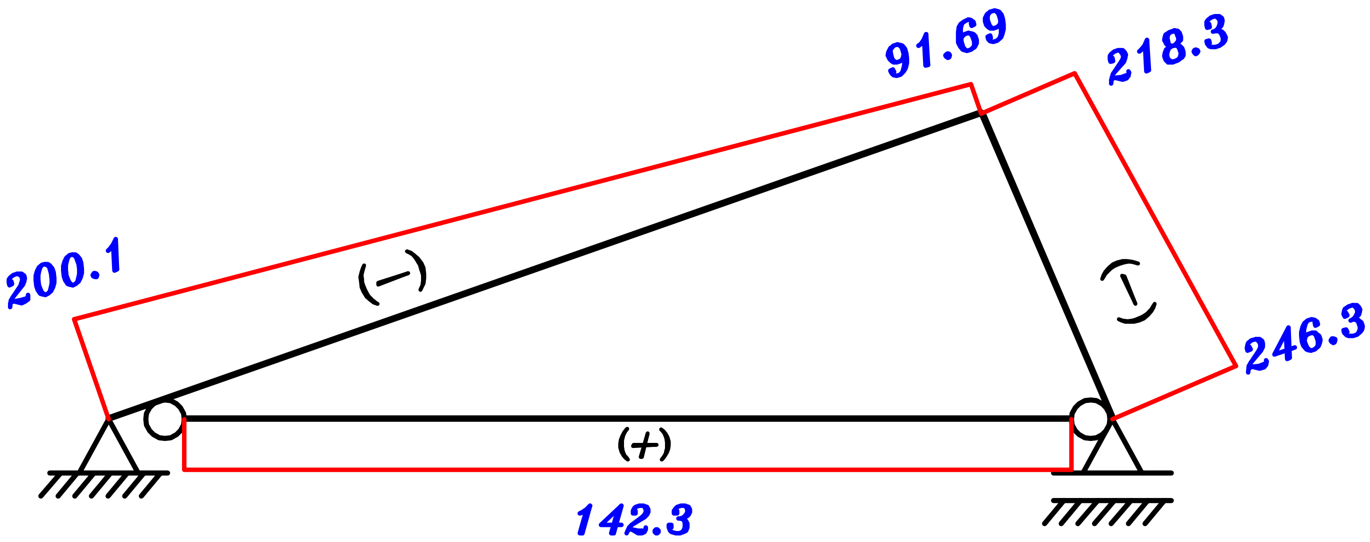
$$\boxed{\delta_{10} + X \delta_{11} = Zero}$$

$$\frac{-4777.48}{E_c I_2} + X * \frac{33.033}{E_c I_2} = Zero \longrightarrow \boxed{X = 144.6 \text{ kN}}$$





B.M.D.

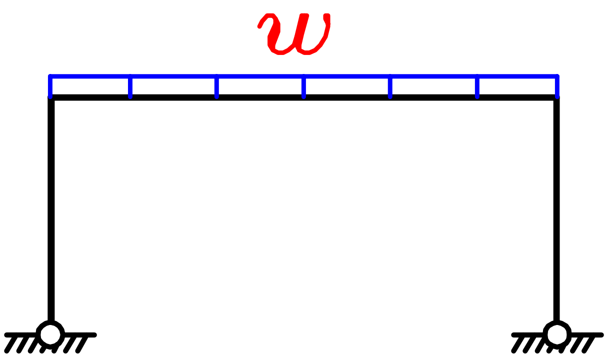


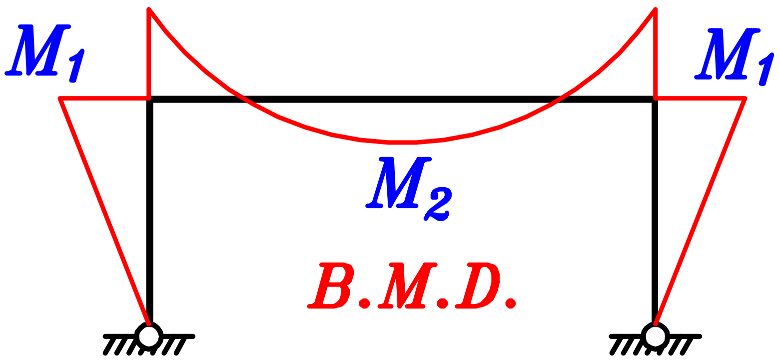
N.F.D.

Thrust Line. (Pressure Line)

تعريف Thrust Line

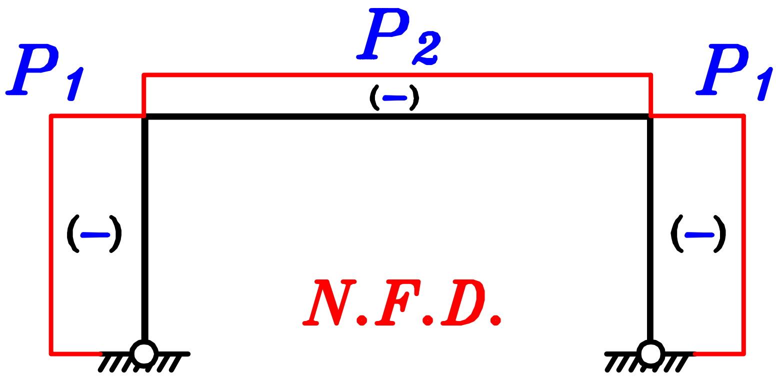
هو عباره عن خط وهمى يوصل بين مكان محصله ال (Normal stresses) الناتجه عن (Axial Force) و (Bending moment) فى جميع قطاعات ال (structure) بحيث اذا كان شكل المنشأ (structure) هو نفس شكل (Thrust Line) سنضمن أن (Axial Force) تؤثر تماما عند محصله كل قطاعات المنشأ و بالتالى لا يوجد (Bending moment) عند كل قطاعات المنشأ (structure).

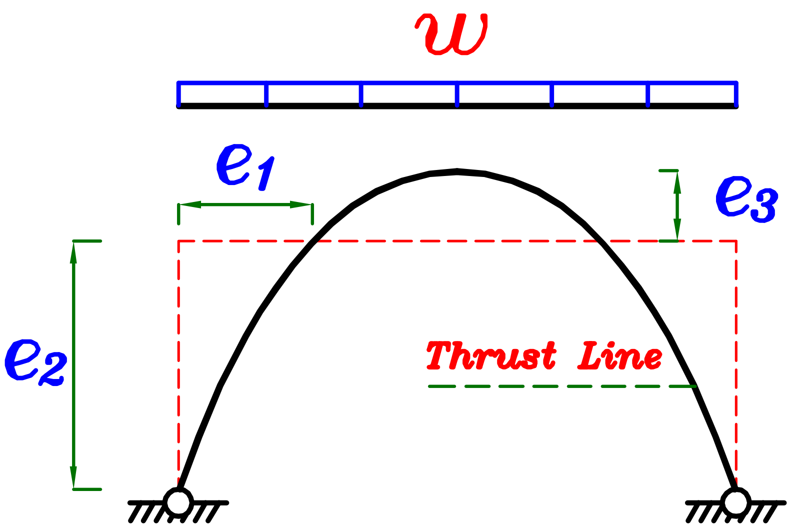




$$e_1 = \frac{M_1}{P_1} \quad , \quad e_2 = \frac{M_1}{P_2}$$

$$e_3 = \frac{M_2}{P_2}$$





اذا تم عمل شكل ال (structure) نفس شكل (Thrust Line) لن يكون هناك (Bending moment) و لكن يوجد فقط (axial Force)

أشهر المنشآت التى يتم عمل شكلها نفس شكل (*Thrust Line*) هى :

1 _ *Triangular Polygon Frame.*

2 _ *Trapezoidal Polygon Frame.*

3 _ *Arch Girder.*

4 _ *Parabolic slab.* (*Arch Slab*).

و لان فى هذه المنشأت تكون قيمه (*axial Force*) **تقريبا** ثابته على جميع القطاعات .

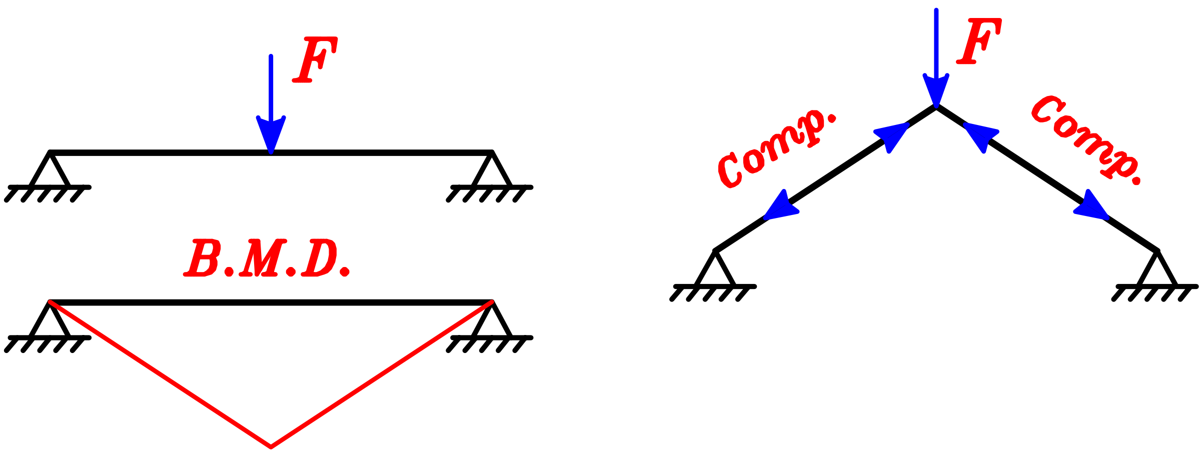
أى أن $\left(e = \frac{M}{P} = \frac{M}{constant} \right)$

لذا اذا رسمنا شكل الـ (*structure*) عكس شكل الـ (*B.M.D.*) يكون هو نفسه شكل الـ (*Thrust Line*) أى لا يكون عليه (*Bending moment*)

و لكن يؤثر عليه فقط (*axial Force*) .

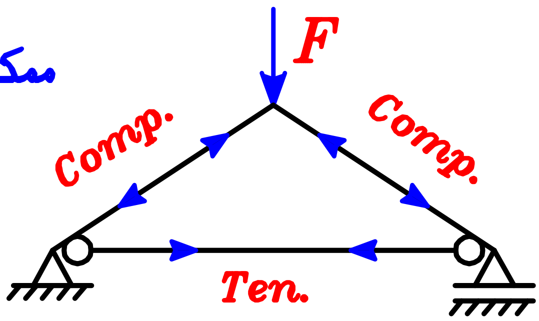
و هذه تعتبر ميزه اقتصاديه لان هذا يوفر فى كميات كلا من الخرسانه و حديد التسليح .

1 _ *Triangular Polygon Frame.*

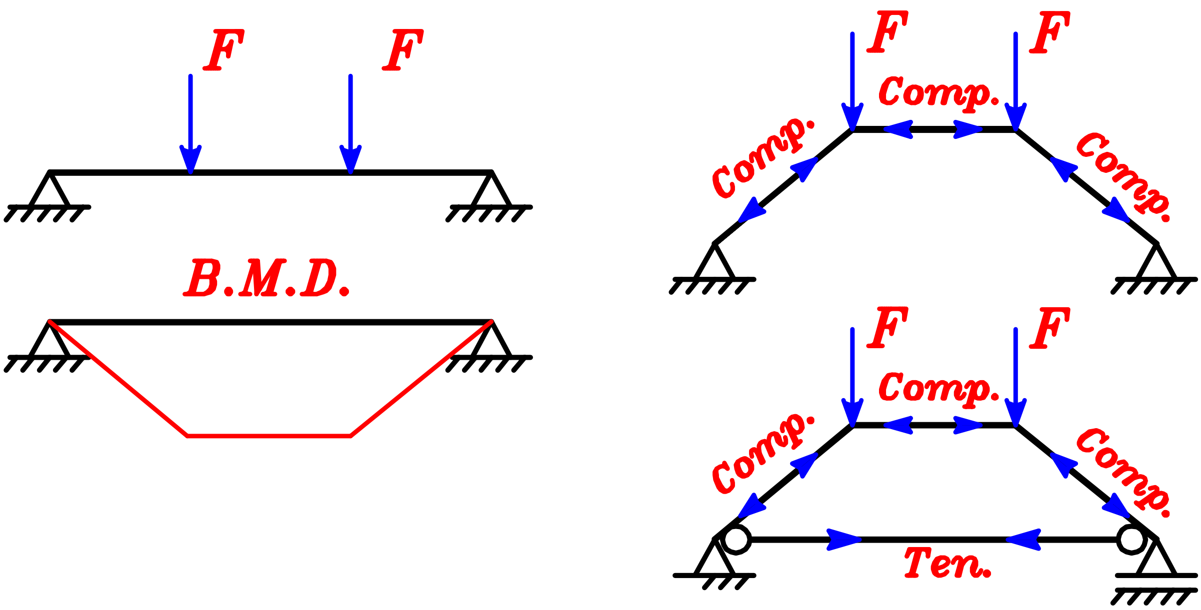


اذا تم عمل الـ (*Girder*) عكس شكل الـ (*moment*) ستكون قيمه الـ (*moment*) عليه تساوى *Zero* و سيؤثر عليه (*axial compression Force*) فقط

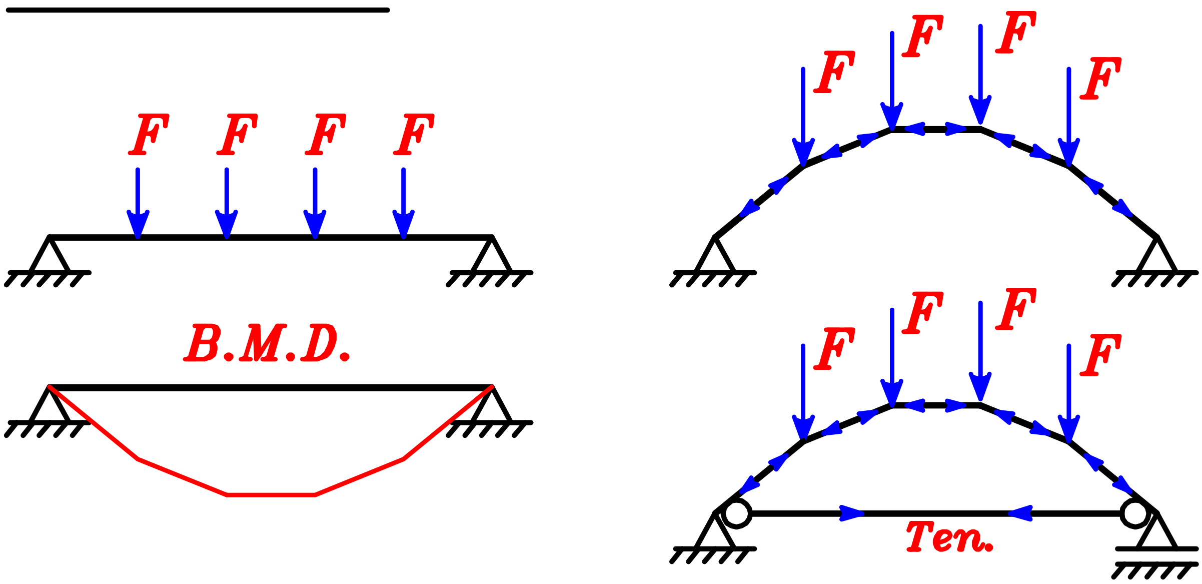
ممكن أخذ الـ (*support*) واحد (*hinge*) و الاخر (*roller*) لكن يتم معها وضع (*link member*) كما بالشكل



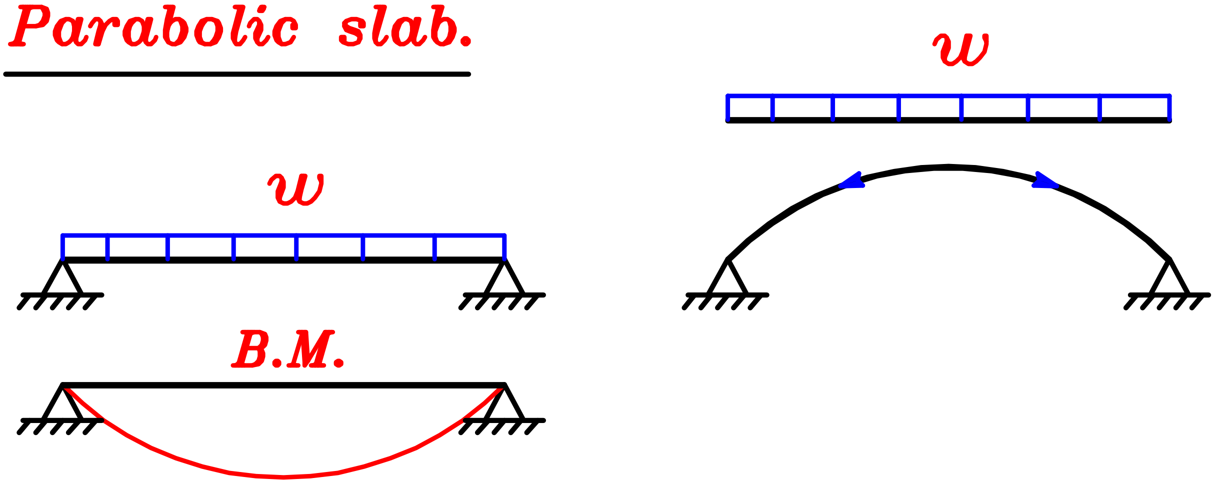
2 – Trapezoidal Polygon Frame.



3 – Arch Girder.



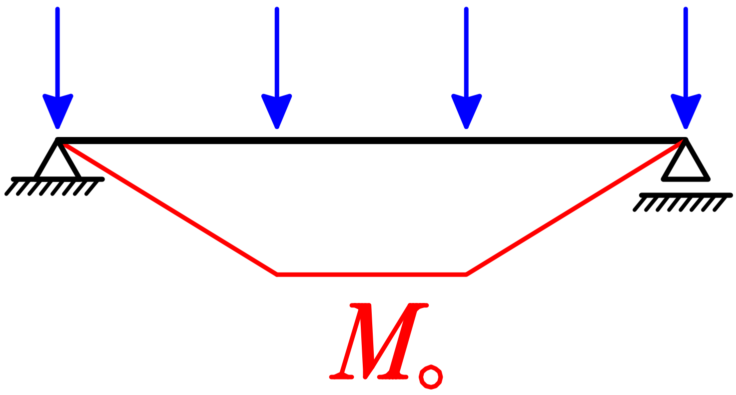
4 – Parabolic slab.

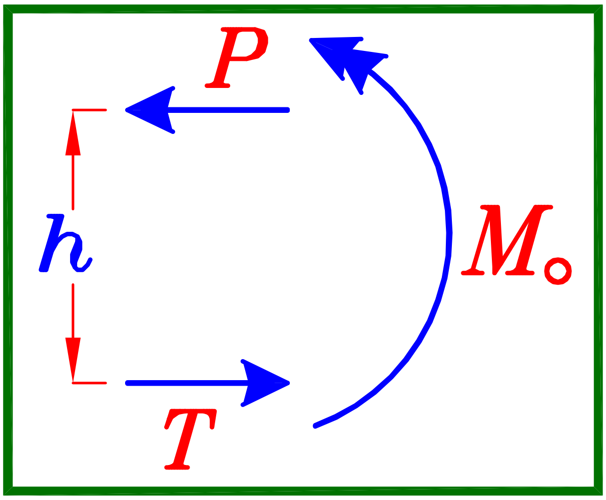


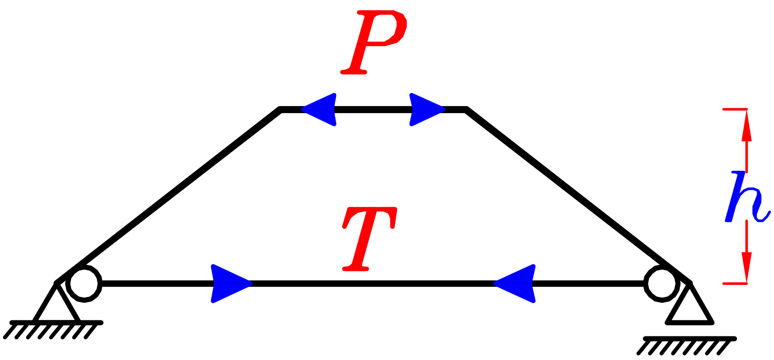
اذا تم عمل ال (*Slab*) عكس شكل ال (*moment*) ستكون قيمة ال (*moment*)

عليه تساوى *Zero* و سيؤثر عليه (*axial compression Force*) فقط

Concept of Polygon Frames.





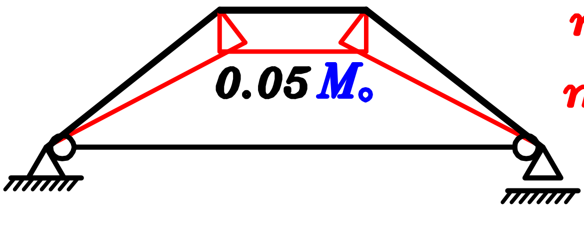


$$P = T = \frac{M_o}{h}$$

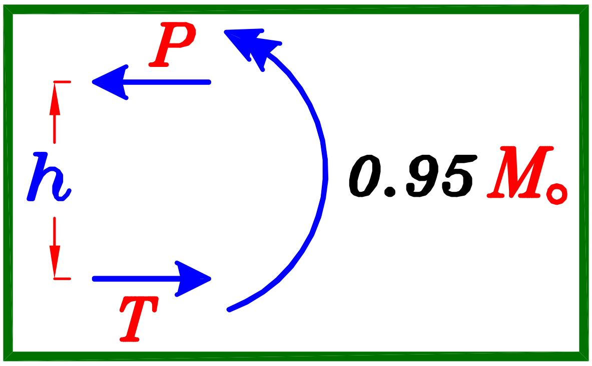
تعتمد فكره ال Polygon Fram على تحويل ال Bending moment الى Couple اى الى Tension Normal Forces & Compression Normal Forces

و ذلك للتوفير لانه عند تصميم قطاع عليه pure Compression ستكون كميه الخرسانه و الحديد قليله مما يعمل على تقليل ثمن ال member

و عند تصميم قطاع عليه pure Tension تكون كميه الحديد كبيره و كميه الخرسانه قليله و تكون ايضا نسبيا ثمن ال member أقل .



اذا حدثت استطاله بسيطه لل Tie سيحدث moment بسيط قيمته فيي حدود $0.05\, M_o$ اذا قيمه ال moment الذى سيتحول لـ couple يساوى تقريبا $0.95\, M_o$

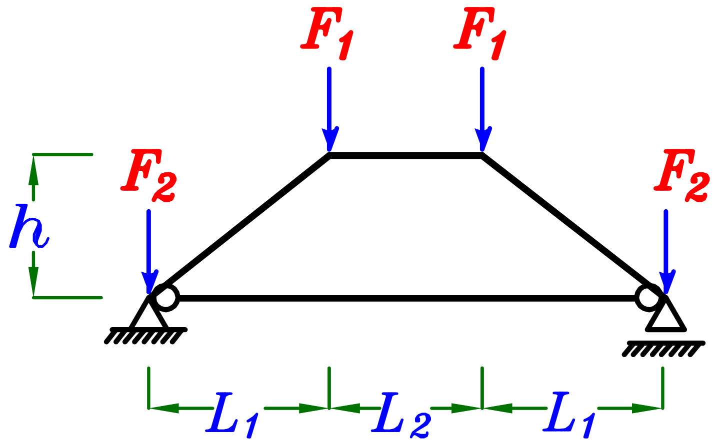


$$P = 0.95\ \frac{M_o}{h}$$

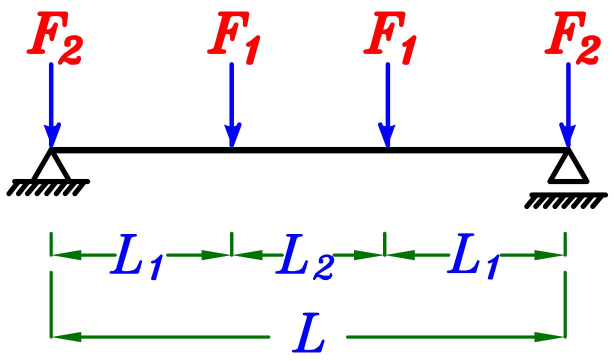
$$T = 0.95\ \frac{M_o}{h}$$

Polygon Frames.

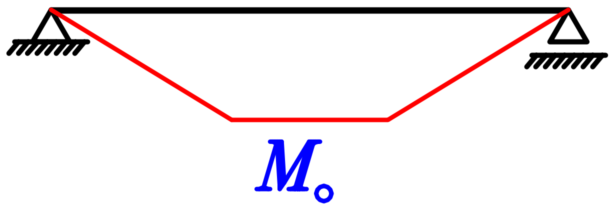
Neglect o.w. of the Frame.



نفرض وجود كمره تخيليه
لها نفس ال span الافقى لل Frame



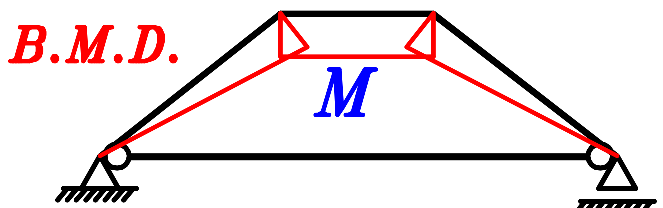
نحسب قيمه أكبر moment
للكمره التخليه و يسمى ((M_o))

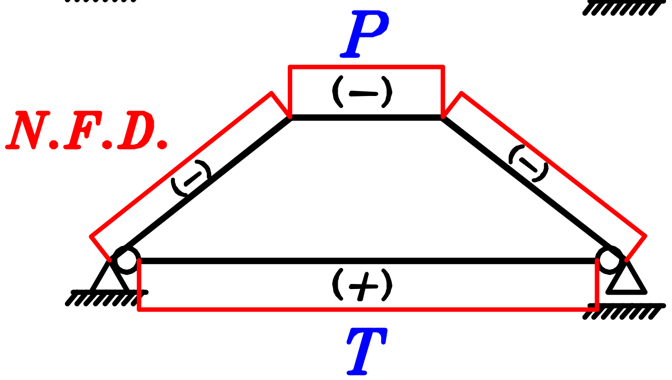


$$\boxed{M = 0.05\ M_o}$$

(From Extension of the Tie)

نتيجه لحدوث استطاله بسيه في ال Tie
يحدث عزم على ال Frame قيمته $0.05\ M_o$

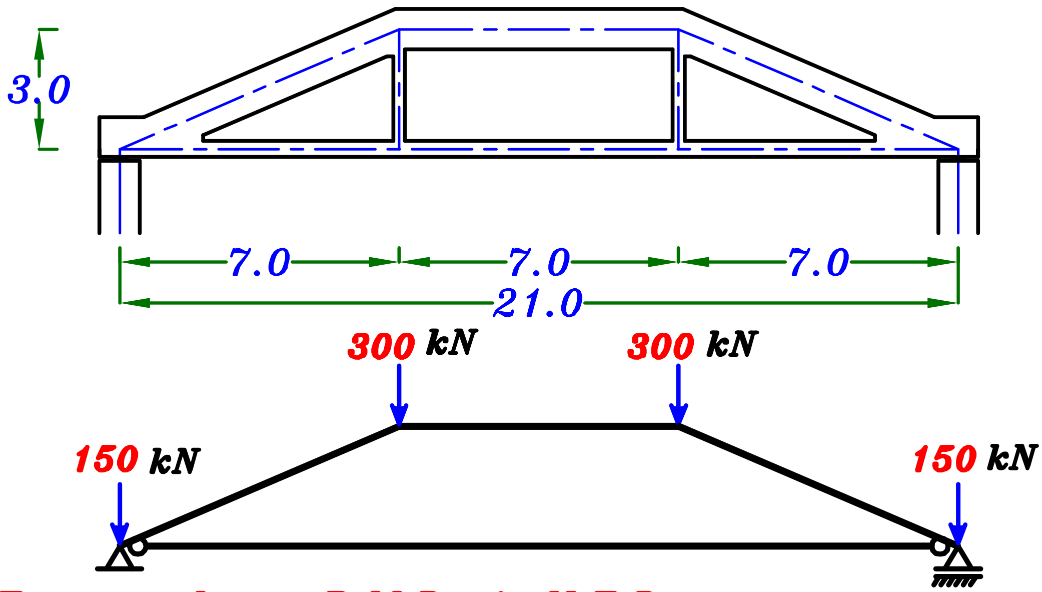




$$\boxed{P = 0.95\ \frac{M_o}{h}}$$

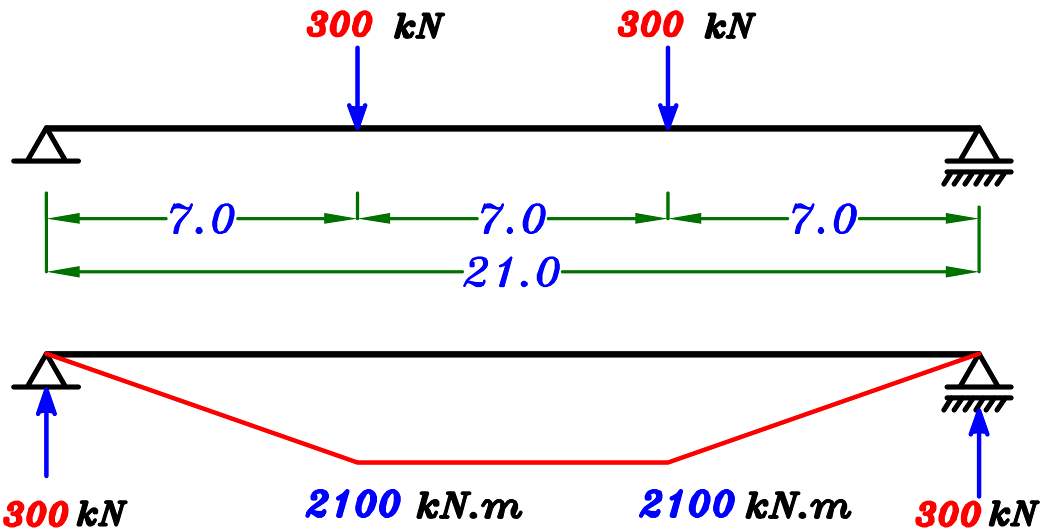
$$\boxed{T = 0.95\ \frac{M_o}{h}}$$

Example.



For the Polygon Frame, draw B.M.D. & N.F.D.

Solution.

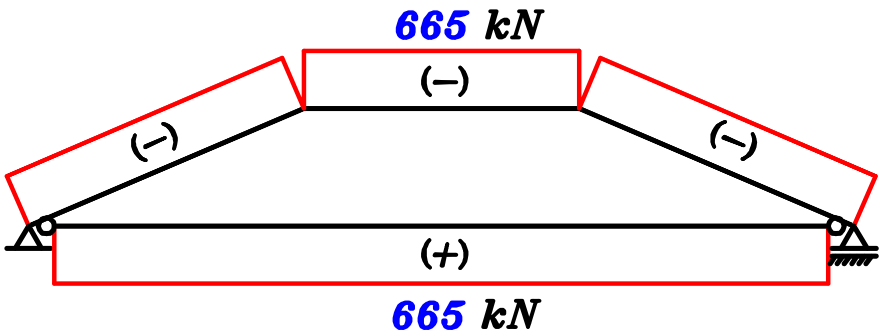


$$M_o = 300 * 7.0 = 2100 \quad kN.m$$

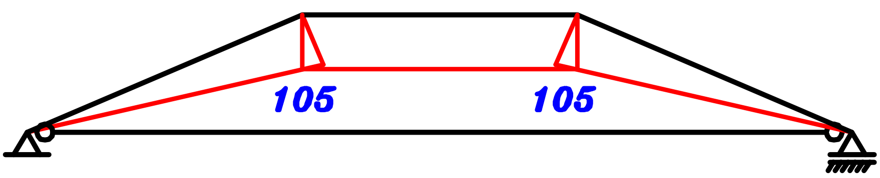
$$M = 0.05\ M_o = 0.05\ (2100) = 105 \quad kN.m$$

$$P = T = 0.95\ \frac{M_o}{h} = 0.95 * \frac{2100}{3.0} = 665\ kN$$

N.F.D.



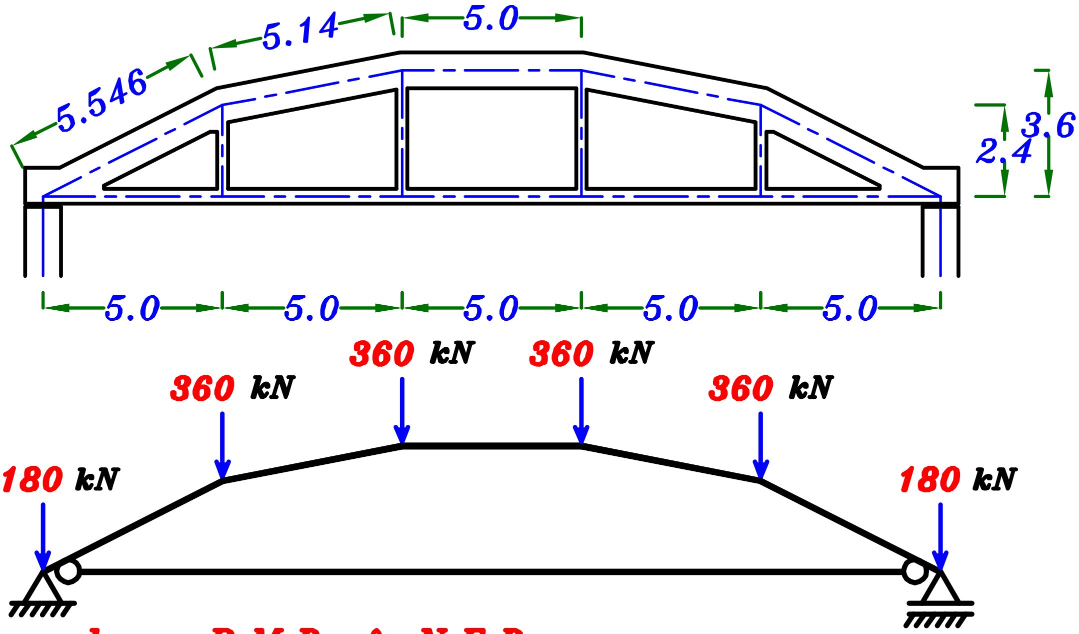
B.M.D.



Arch Girder. the same as polygon Frames.

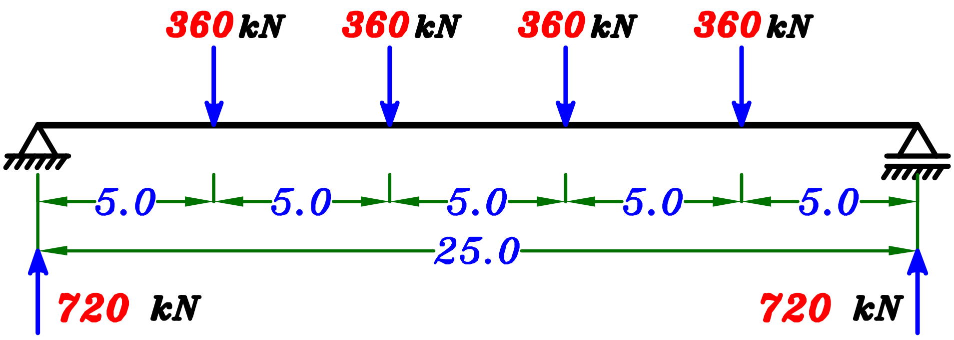
Solving by Approximate Method.

Example.

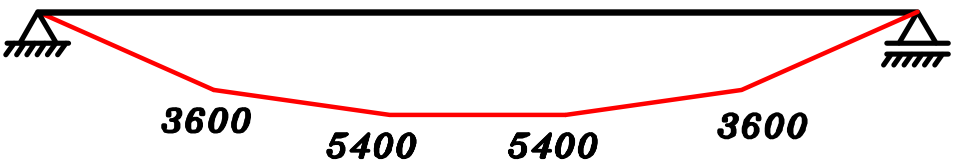


For the Arch Girder, draw B.M.D. & N.F.D.

Solution.



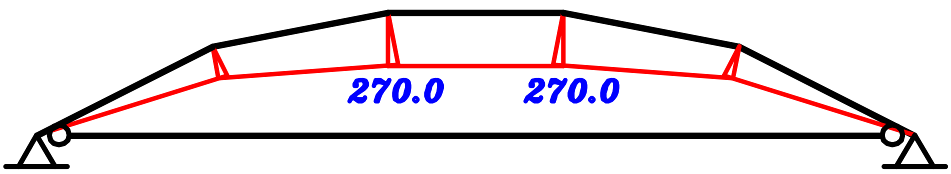
$M_o = 5400$ kN.m



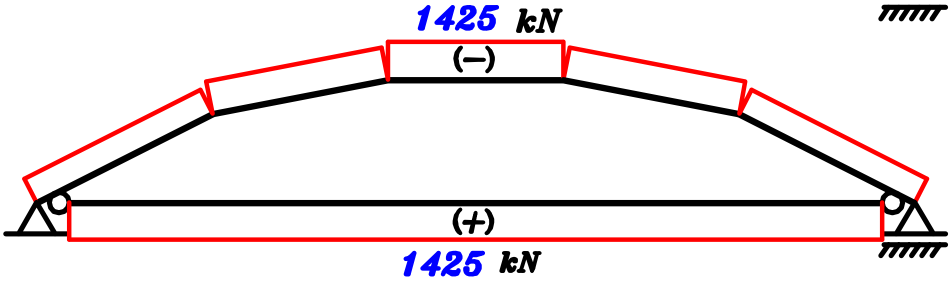
$$M = 0.05\ M_o = 0.05\ (5400) = 270 \text{ kN.m}$$

$$P = T = 0.95\ \frac{M_o}{h} = 0.95 * \frac{5400}{3.60} = 1425 \text{ kN}$$

B.M.D.



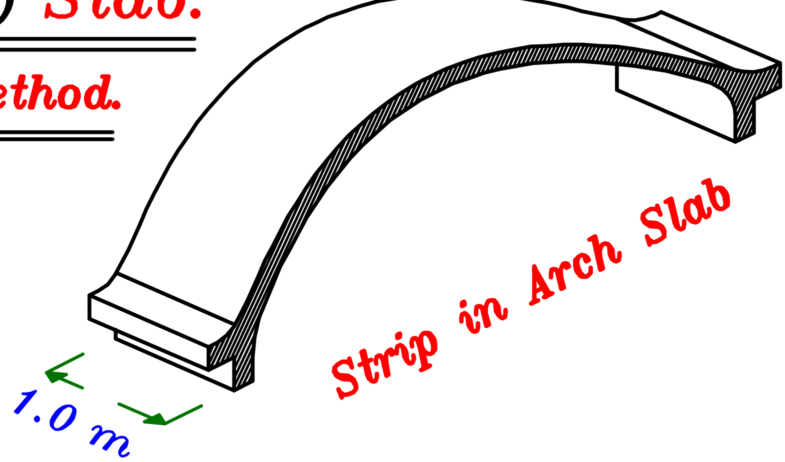
N.F.D.



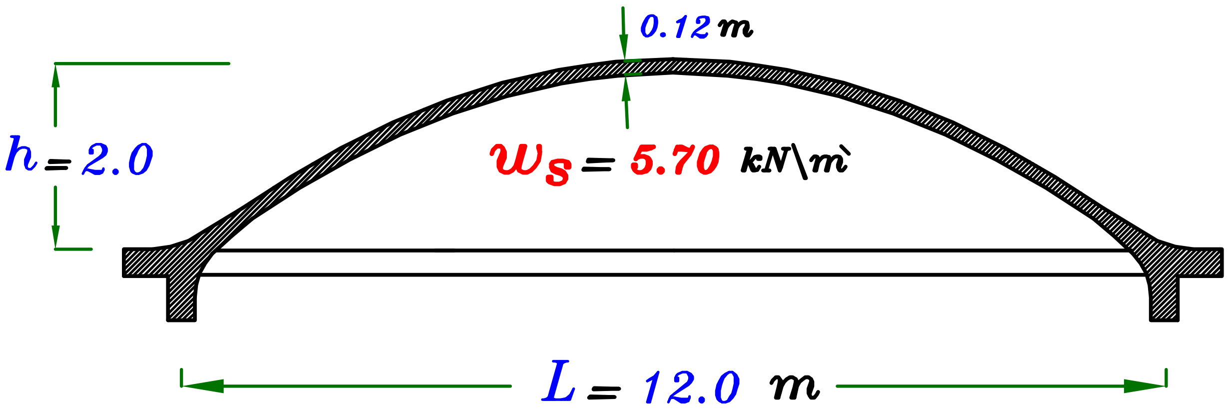
Parabolic (Arch) Slab.

Solving by Approximate Method.

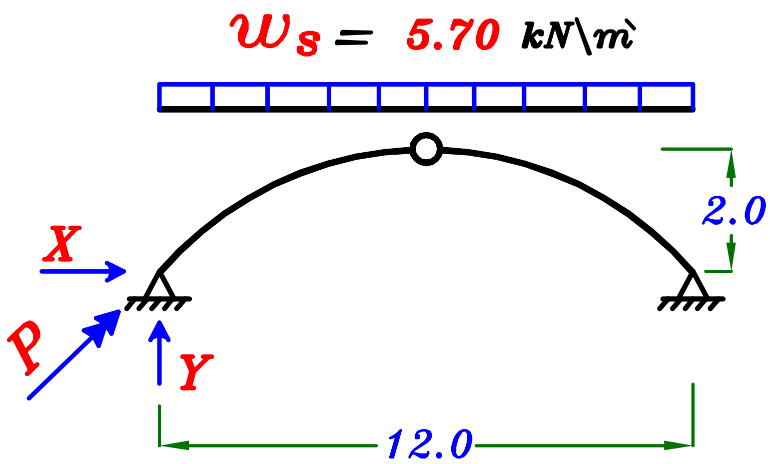
Example.



For the Arch Slab Calculate N.F.



Solution.



M = Zero

To Get N.F.

$$Y = \frac{w\,L}{2} = \frac{5.70 * 12}{2} = 34.2 \ kN\backslash m$$

$$X = \frac{w\,L^2}{8\,h} = \frac{5.70 * 12^2}{8 * 2.0} = 51.3 \ kN\backslash m$$

$$P = \sqrt{X^2 + Y^2} = \sqrt{34.2^2 + 51.3^2} = 61.65 \ kN$$

Moment of Inertia For T-Sec.

$$\left.\begin{matrix} \dfrac{t_s}{t} \\[2ex] \dfrac{b_o}{B} \end{matrix}\right\} \rightarrow \mu \rightarrow \boxed{I = (\mu * 10^{-4})\, B\, t^3}$$

Moment of Inertia of T- Sections :

$$I_s = \mu B t^3$$

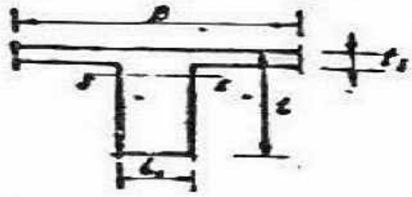


Table of μ-values x 10^{-4} : $\boxed{I}$

$\frac{b_o}{B}$	t_s/t										
	.05	.10	.15	.20	.25	.30	.35	.40	.50	.55	.60
.05	97	109	111	111	112	115	122	132	169	196	231
.06	110	125	129	129	129	132	137	147	181	207	241
.07	122	140	145	146	146	148	152	161	193	218	251
.08	133	154	161	162	162	163	167	175	205	229	260
.09	143	167	176	178	178	178	182	189	217	240	270
.10	154	179	190	192	192	193	196	202	228	250	279
.11	164	192	203	206	207	207	209	215	240	260	[illegible]
.12	173	204	216	220	221	221	223	227	251	271	[illegible]
.13	182	215	229	233	234	234	236	240	262	281	307
.14	191	226	241	246	247	247	248	252	272	290	316
.15	200	236	252	258	260	260	261	264	283	300	324
.16	209	245	263	270	272	272	273	276	293	310	333
.17	217	255	273	282	284	284	285	287	304	319	342
.18	225	265	284	293	296	296	296	298	314	329	[illegible]
.19	234	274	295	304	307	308	307	309	324	338	[illegible]
.20	242	283	304	314	318	319	319	320	333	347	367
.22	258	301	323	334	339	340	340	341	353	365	[illegible]
.24	275	318	342	354	359	360	360	361	371	382	400
.26	291	334	360	373	378	380	380	381	389	399	417
.28	306	350	376	390	397	399	399	400	407	416	431
.30	320	366	392	407	415	417	418	418	424	432	440
.32	336	380	408	424	432	435	435	435	441	446	461
.34	352	395	424	440	445	452	452	452	457	464	475
.36	367	410	438	455	464	468	469	469	473	479	490
.38	382	426	453	470	480	484	485	485	488	497	504
.40	397	441	466	485	495	499	500	500	503	508	517
.42	412	454	482	499	509	514	515	515	518	522	530
.44	427	468	496	513	523	528	530	530	532	536	[illegible]
.46	441	482	509	527	537	542	544	544	546	549	557
.48	456	496	523	540	551	556	558	558	560	563	[illegible]
.50	470	509	533	553	564	569	571	572	573	576	582
.55	505	544	567	585	596	601	604	604	605	607	612
.60	544	575	599	616	626	631	634	635	636	637	641
.65	561	609	630	645	655	650	663	654	654	655	668
.70	616	642	660	674	653	688	691	691	692	692	695
.75	652	675	691	702	709	714	717	718	716	718	720
.80	689	706	720	729	736	[illegible]	742	743	743	743	744
.90	761	770	779	782	786	785	789	790	790	790	791
1.0	833	833	833	833	833	833	833	833	833	833	833